An Evolutionary Algorithm with Heuristic Longest Cycle Crossover for Solving the Capacitated Vehicle Routing Problem

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2019 IEEE CONGRESS ON EVOLUTIONARY COMPUTATION, WELLINGTON, NEW ZEALAND, 10-13 JUNE 2019

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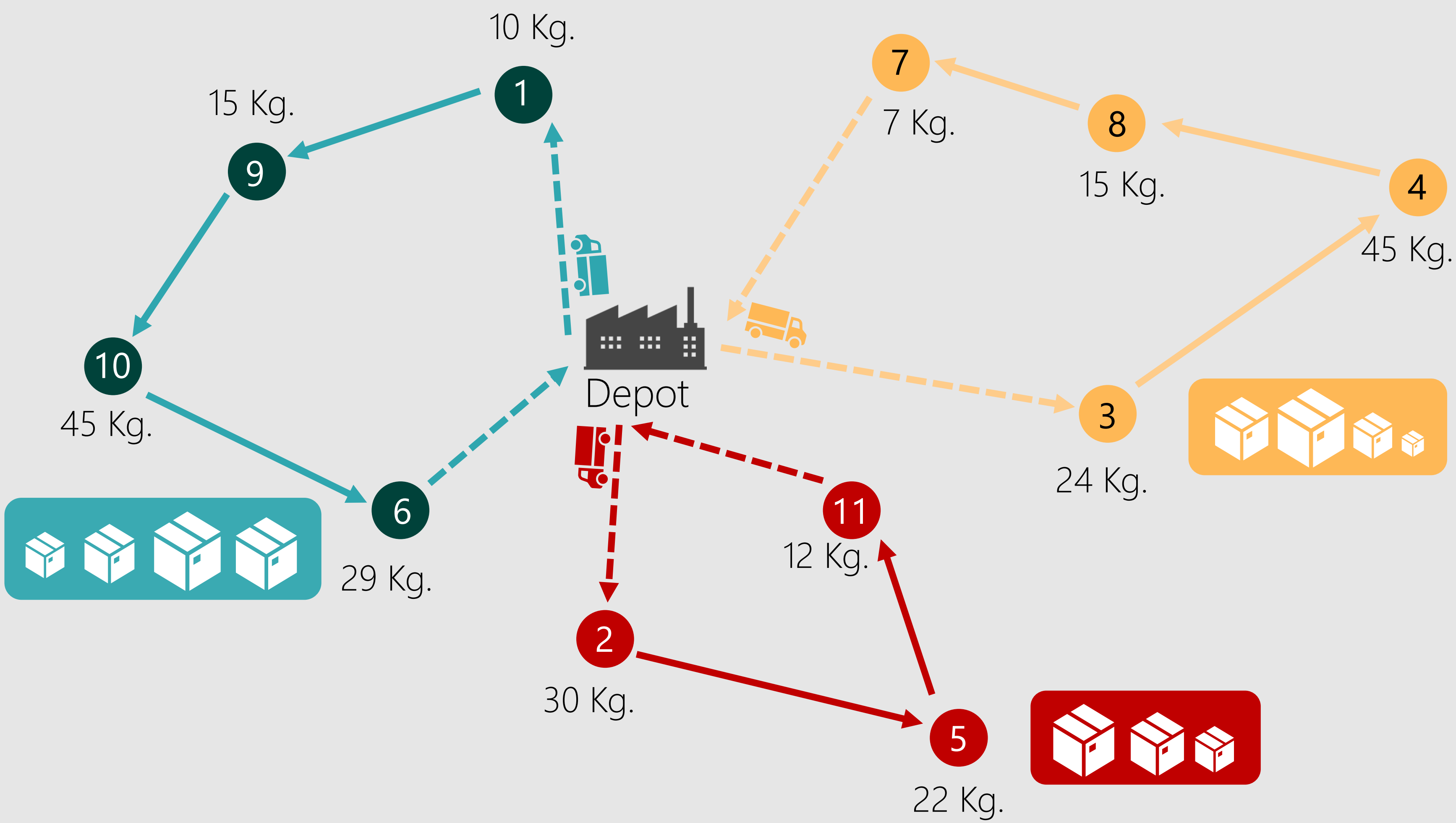
- CVRP: The Problem Introduction

(CVRP: Capacitated Vehicle Routing Problem)



X 3

Minimize travel distance

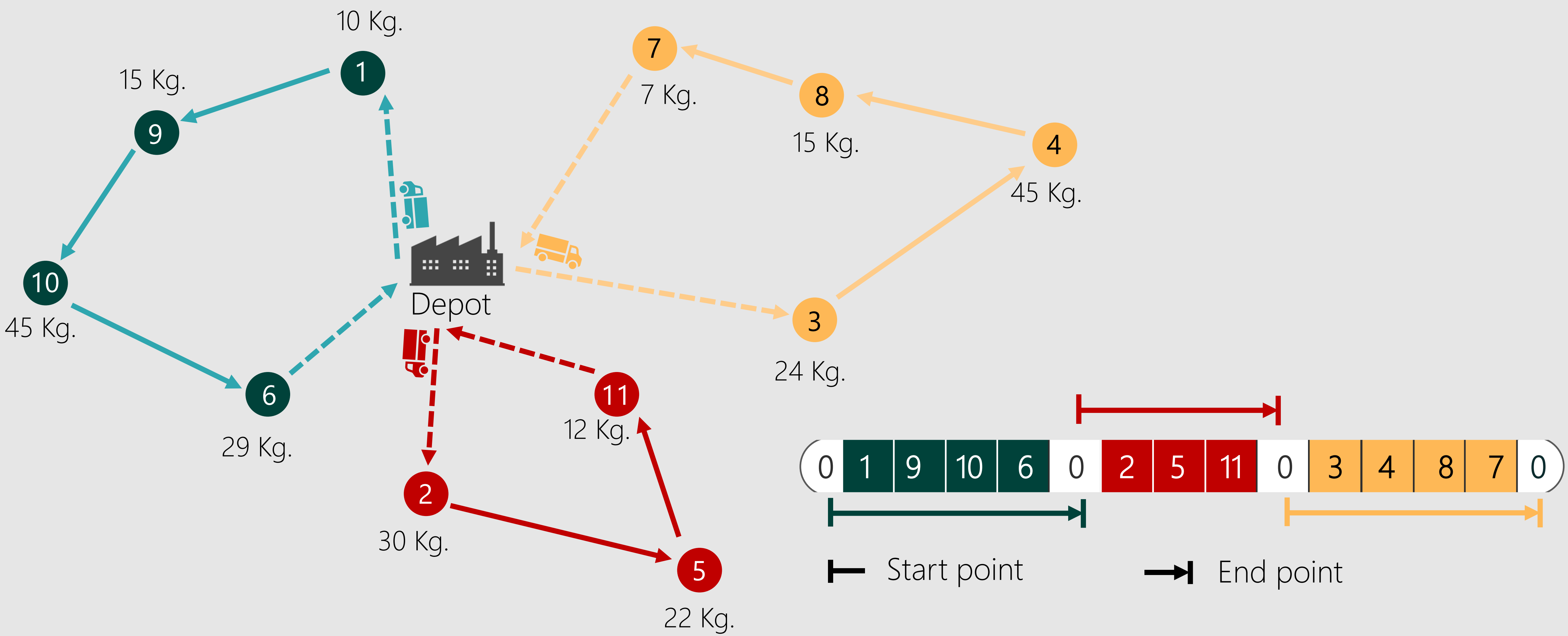
CVRP's constraints

Customer
- Customer visited only one time

Vehicle
- No over load
- No extra vehicle

CVRP: The Problem Introduction

(CVRP: Capacitated Vehicle Routing Problem)



Introduction

- Brief Literature Review

Problem	Algorithms
CVRP	Tabu Search Algorithm
	Simulated Annealing Algorithm
	Evolutionary Algorithm (EA)
	Ant Colony Optimization
	Artificial Bee Colony Algorithm

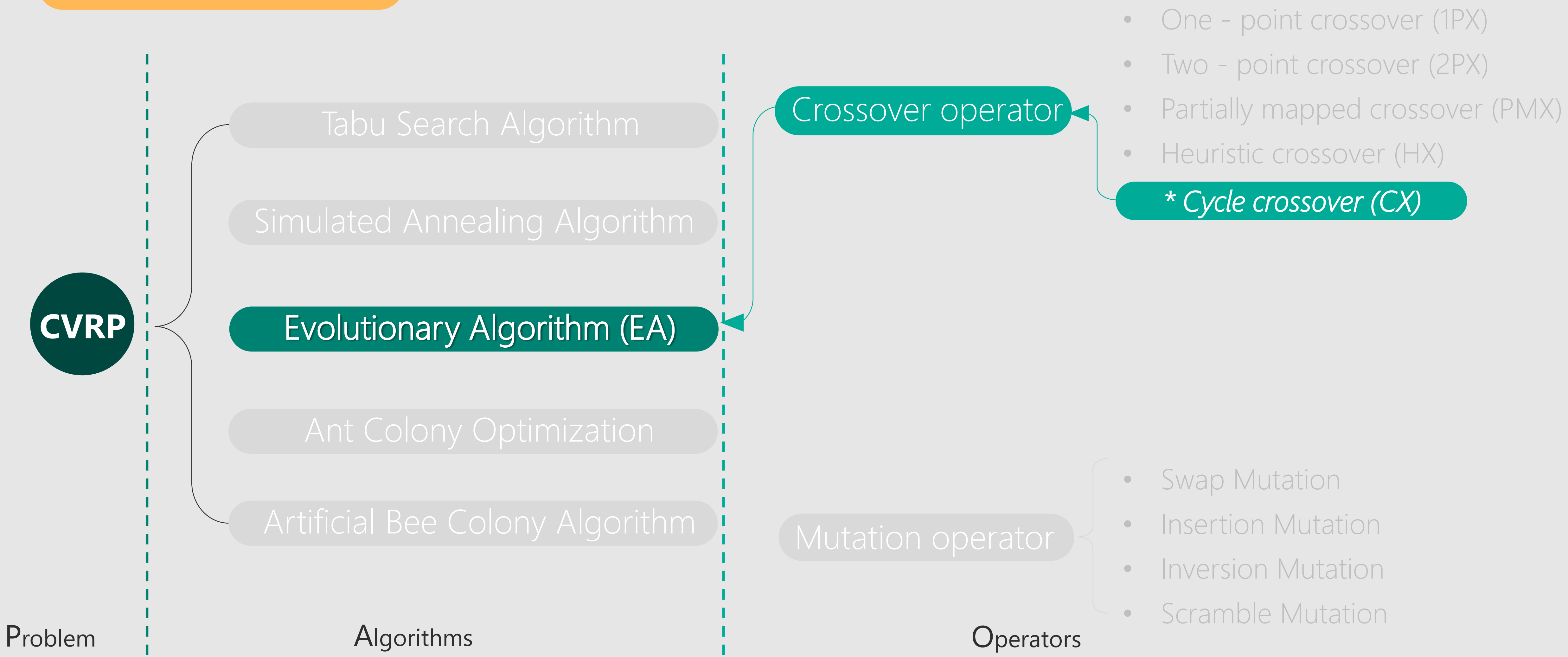
Brief Literature Review

CVRP

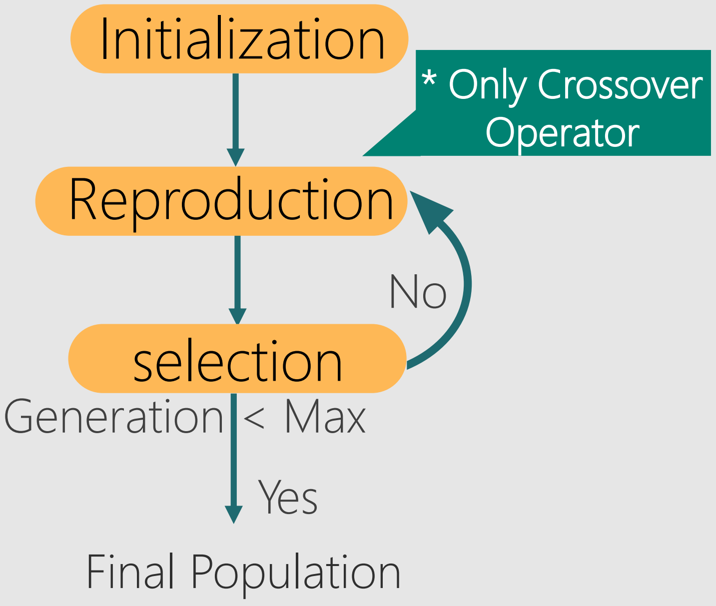
- Tabu Search Algorithm
- Simulated Annealing Algorithm
- **Evolutionary Algorithm (EA)**
  - Crossover operator
    - One - point crossover (1PX)
    - Two - point crossover (2PX)
    - Partially mapped crossover (PMX)
    - Heuristic crossover (HX)
    - Cycle crossover (CX)
  - Mutation operator
    - Swap Mutation
    - Insertion Mutation
    - Inversion Mutation
    - Scramble Mutation
- Ant Colony Optimization
- Artificial Bee Colony Algorithm

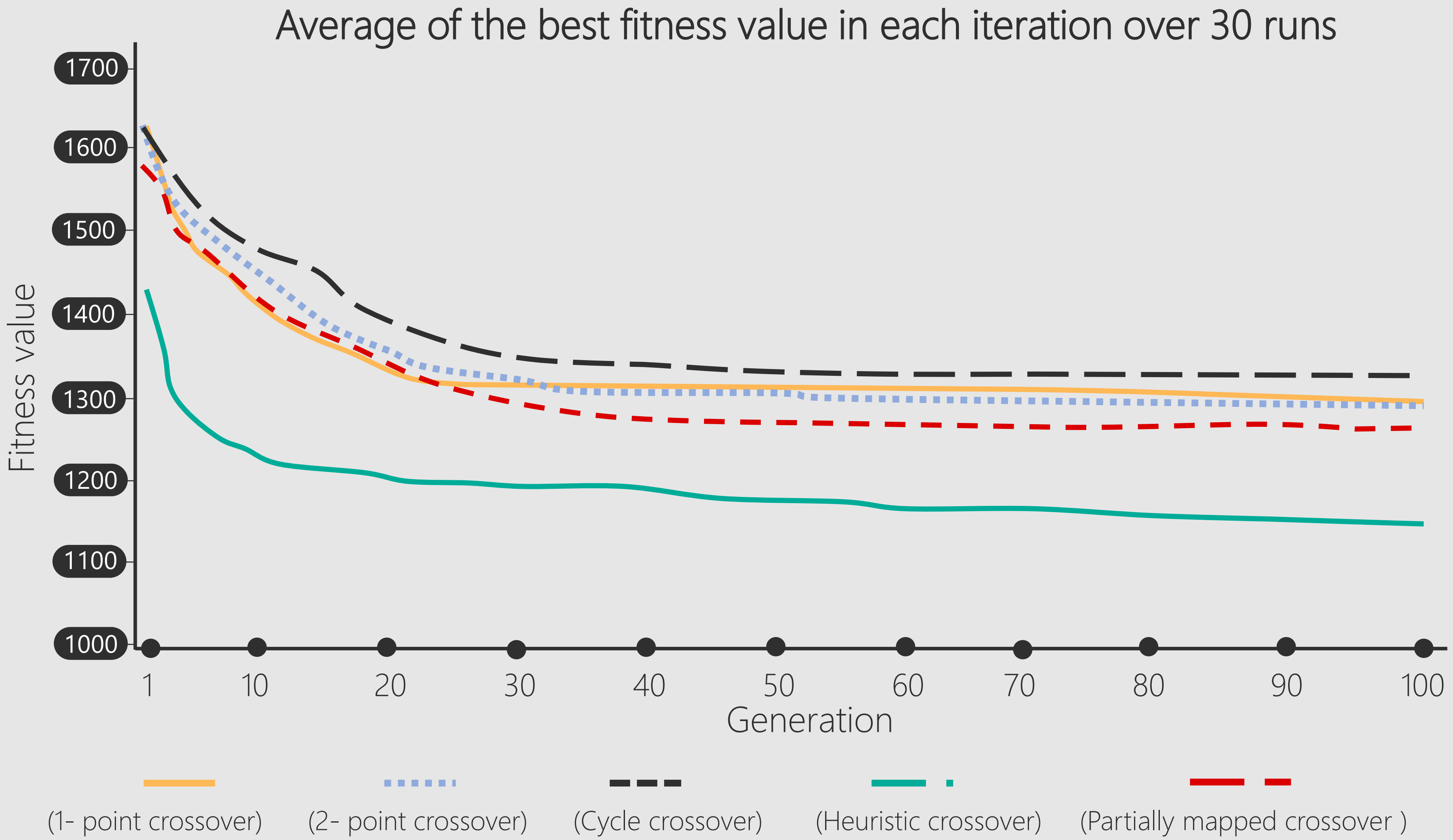
Problem | Algorithms

Brief Literature Review



The Crossover Operator's Performance





Strategies for Improvement

Longest cycle selection

Create large difference between parents and offspring.

Nearest neighbor heuristic

Keep short travel distance during the big change.

CX — Cycle Crossover → HLCX — Heuristic Longest Cycle Crossover

The EA's Mechanism

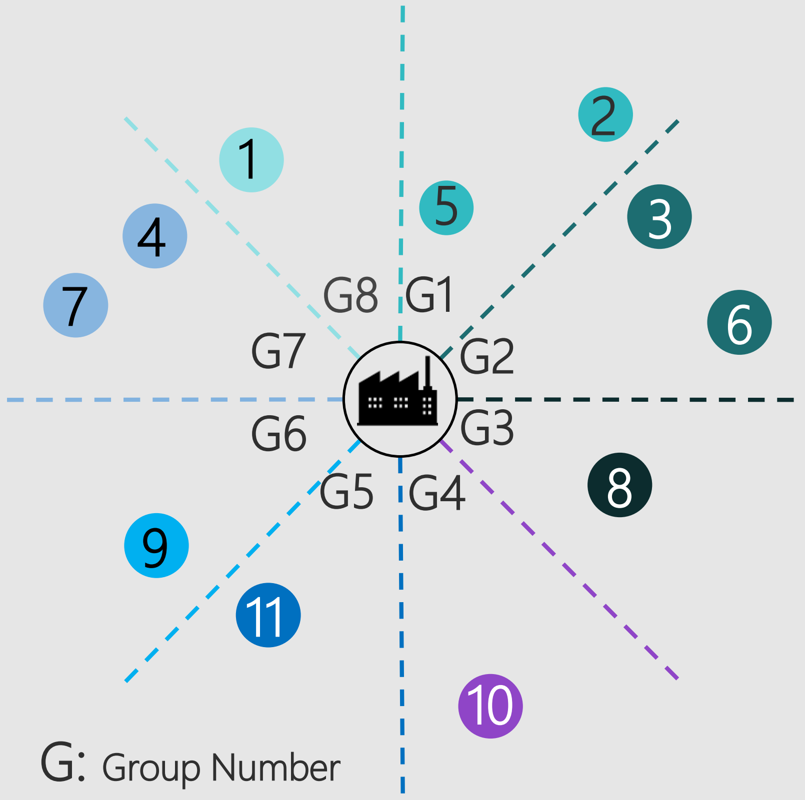
Initialization — Encoding → Decoding

Reproduction — Duplicate Removal → Tournament Selection → ***Crossover*** → Local Refinement

selection — Selection→ Mutation → 2-opt

Generation < Max: No → Reproduction; Yes → Final Population

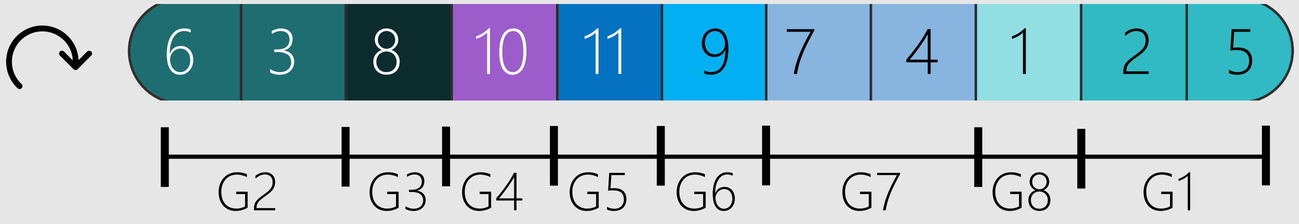
The EA's Mechanism (Encoding)



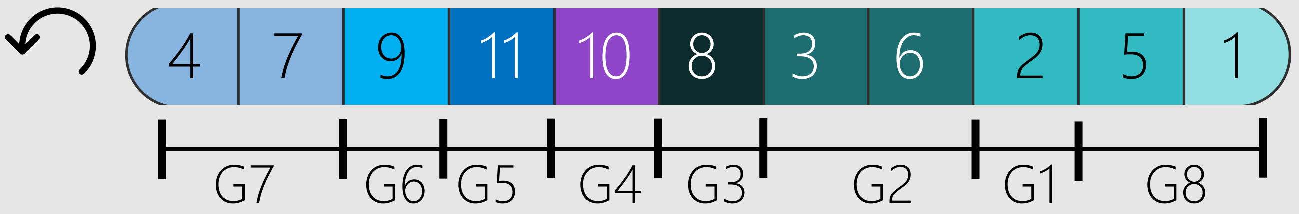
G: Group Number

- Random the 1st group to put inside customer sequence, then move to the next closet group and **NP/2 will created following clockwise and the other counterclockwise.**
- The order inside each can be arrange randomly.

clockwise



counterclockwise



Initialization
- Encoding
- Decoding

Reproduction
- Duplicate Removal
- Tournament Selection
- Crossover
- Local Refinement

Selection
- selection
- Mutation
- 2-opt

• The EA's Mechanism

(Decoding: Vehicle assignment)

1	4	7	9	11	10	8	3	6	5	2
15	45	8	18	12	75	25	24	48	12	6

X 3 (Maximum capacity : 100 Kg)

Starts with the 1st vehicle, check through customer sequence and select the customer which do not make the total demand violate the maximum capacity till the vehicle cannot serve any customer else.

1	4	7	9	11	-	-	-	-	-	-

15+45+8+18+12 = 98

{1,4,7,9,11} ∈

-	-	-	-	-	10	8	-	-	-	-

75 + 25= 98

{10,8} ∈

-	-	-	-	-	-	-	3	6	5	2

24+48+12+6 = 90

{3,6,5,2} ∈

Initialization
- Encoding
- Decoding

Reproduction
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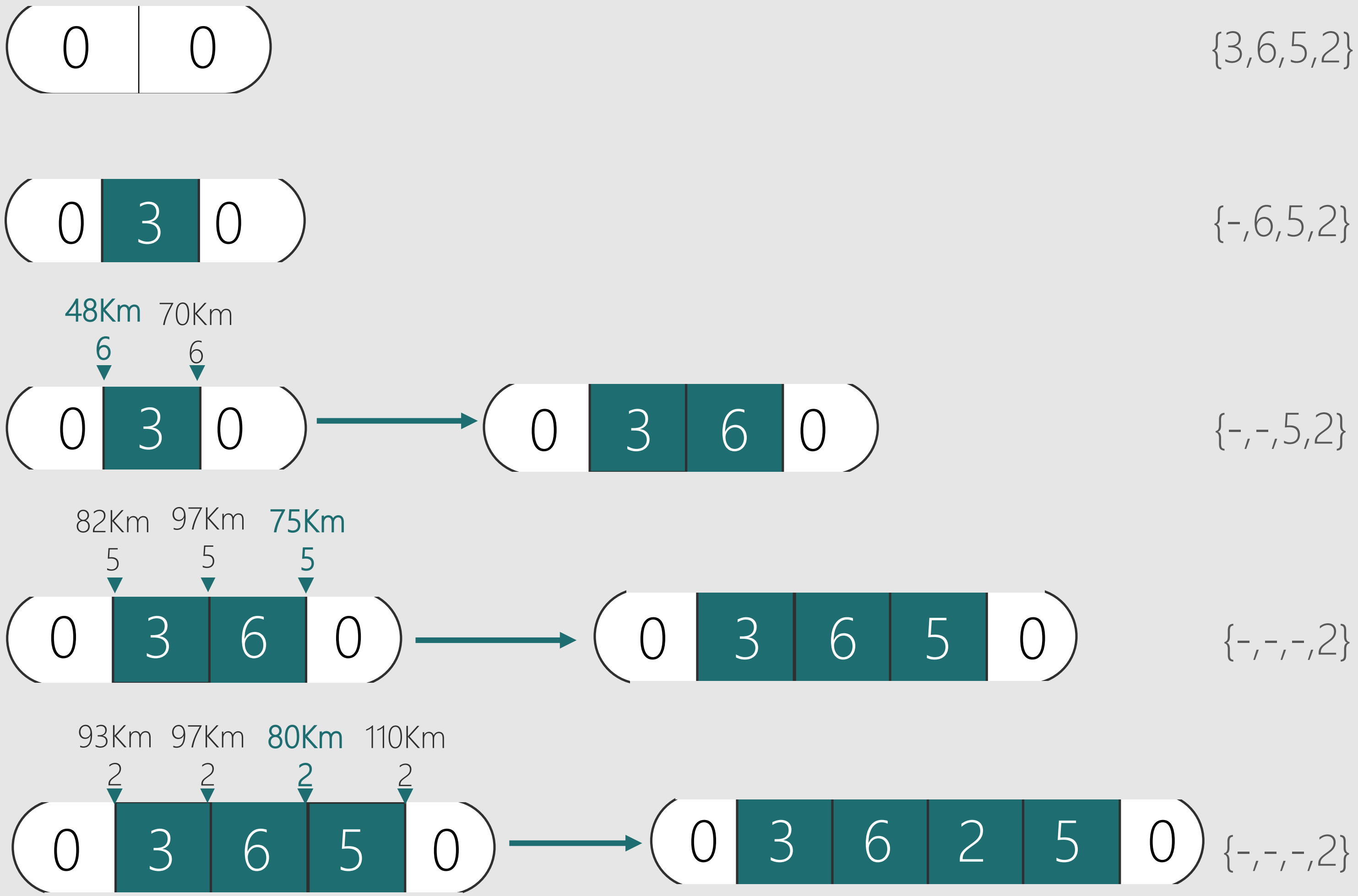
Selection
- selection
- Mutation
- 2-opt

The EA's Mechanism

(Decoding: Greedy insertion heuristic)



Initialization
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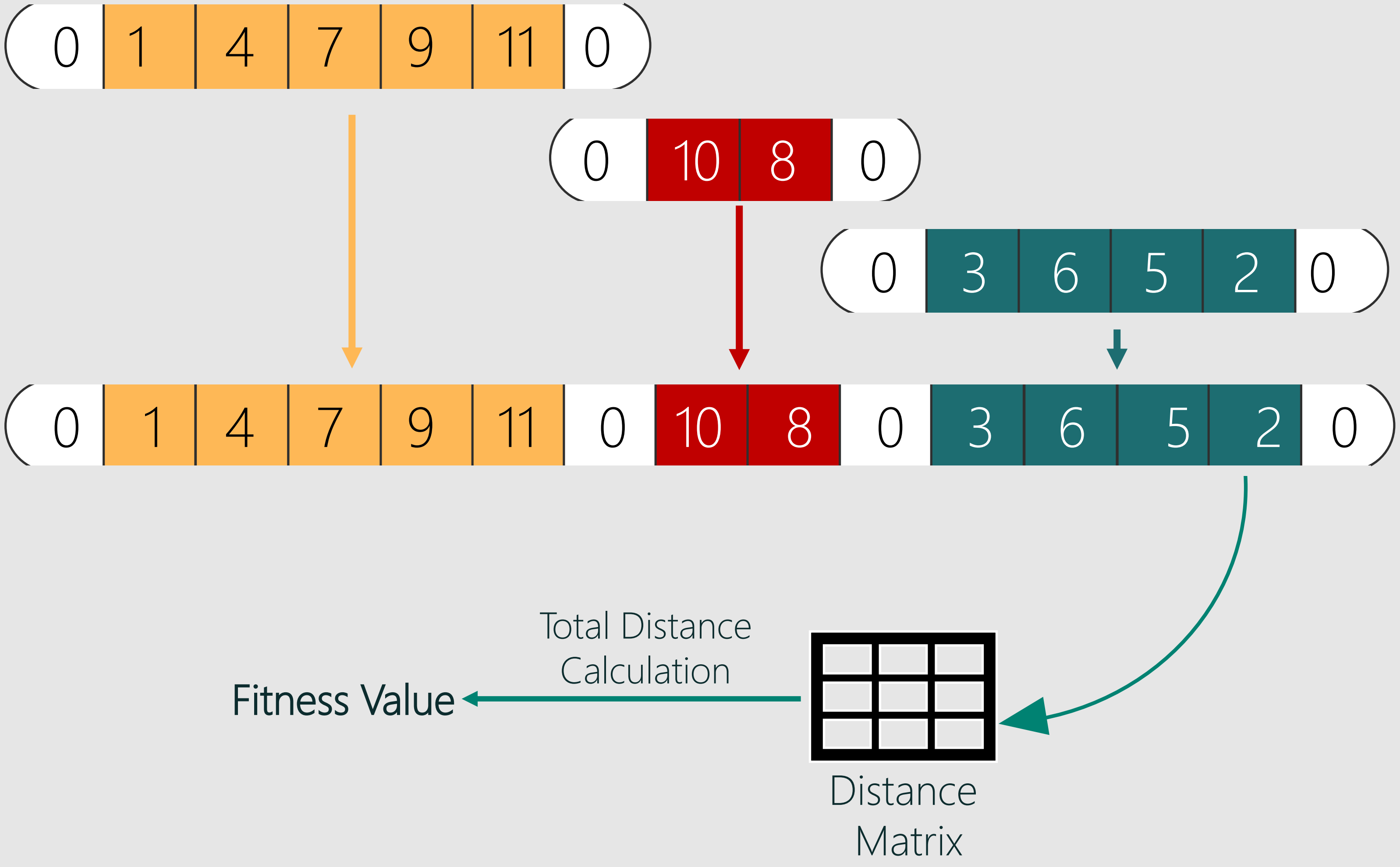
The EA's Mechanism (Decoding)



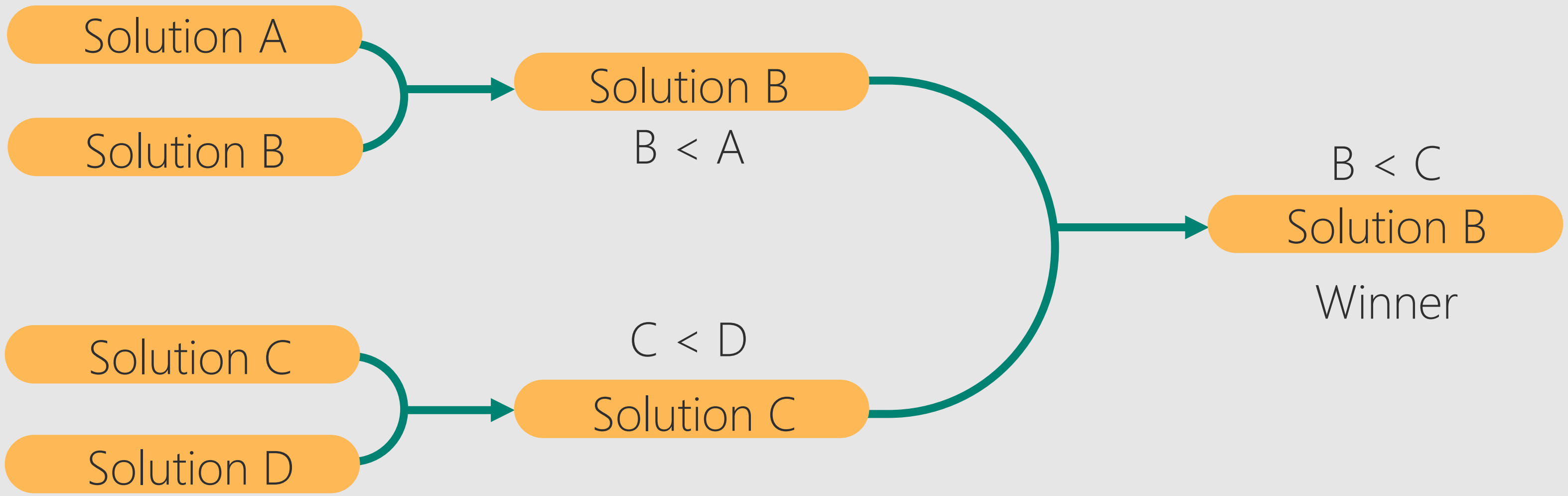
The EA's Mechanism

- Duplicate Removal

Solution 1	205
Solution 2	205
Solution 3	218
⋮	⋮
Solution 100	530

Remove solution 2 ← New Random Solution

- 4-Tournament Selection



Initialization
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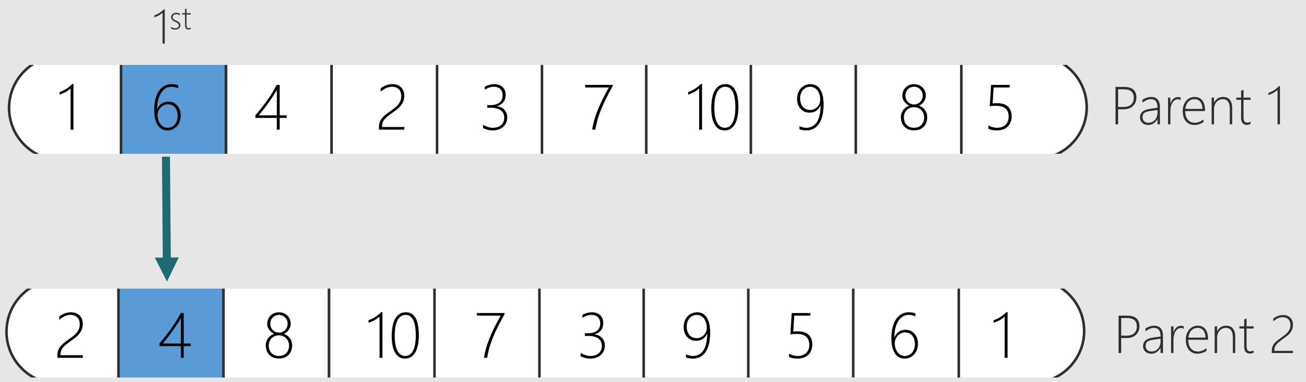
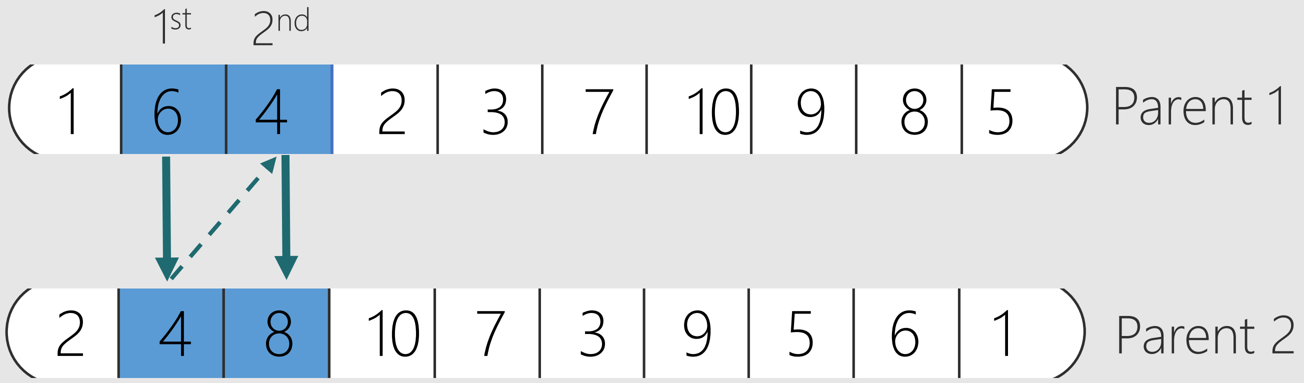
The EA's Mechanism (Original Cycle Crossover)

1. Start with the first unassigned customer in Parent 1 and drop down to the same position in Parent 2.

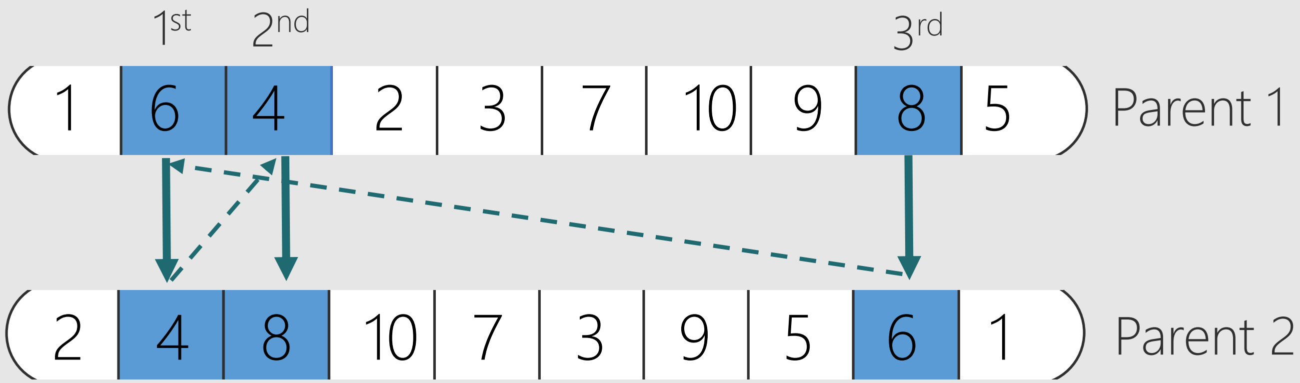


2. Then, look for customer 4 in Parent 1 and drop down to the same position in Parent 2.

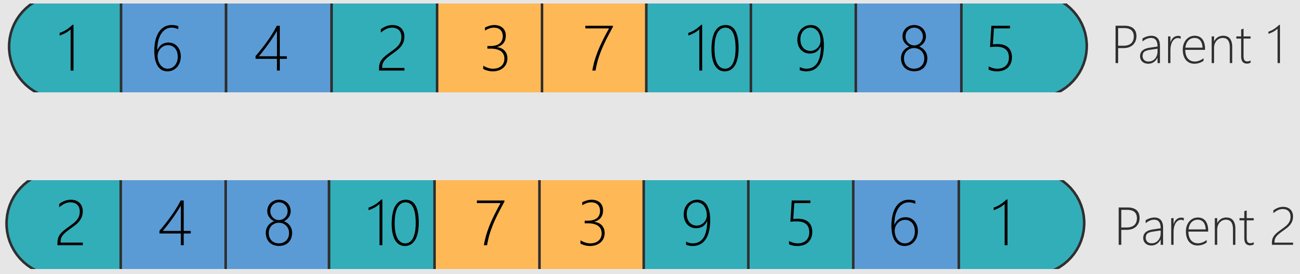


The EA's Mechanism (Original Cycle Crossover)

3. Then, look for customer 8 in Parent 1 and drop down to the same position in Parent 2, the process will be terminated when the 1st customer is found in the Parent 2

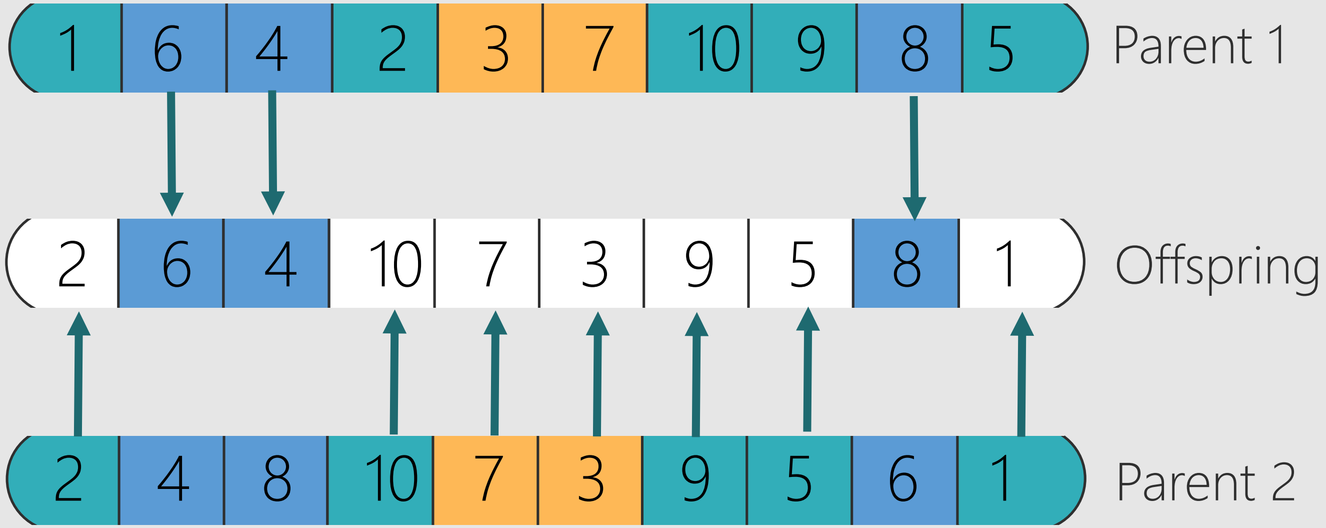


4. Continue the process till all of customer are assigned to their cycle



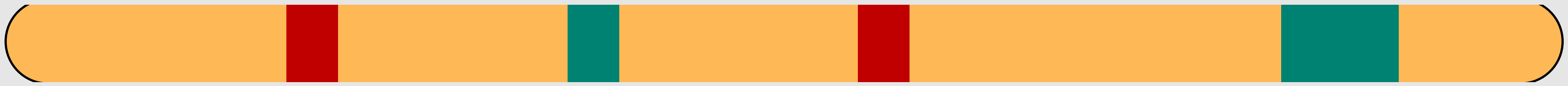
The EA's Mechanism (Original Cycle Crossover)

5. Copy the Blue part from Parent 1 and Green and Yellow from Parent 2



The EA's Mechanism (HLCX VS CX)

- Longest cycle selection
  - By using random selection strategy to select the cycle which have a few customer from one parent and it may lead the original CX to select some small group in some situation. This may lead slow progress

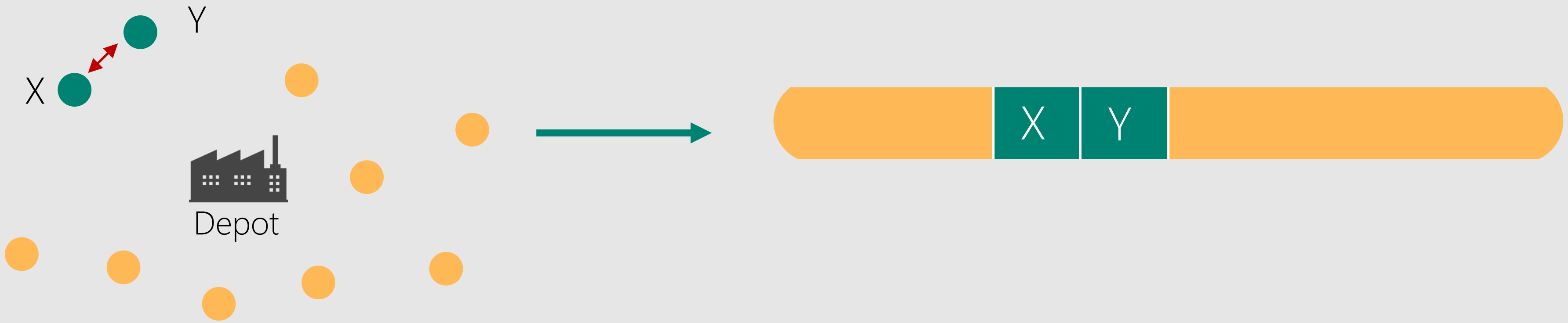


The selected cycle

  - From our preliminary experiment, we found that by selecting only largest can improve the search performance of the original CX

The EA's Mechanism (HLCX VS CX)

- Nearest neighbor heuristic
    - As strategy of nearest neighbor heuristic will attempt to arrange the group of customers who are close in geography located close in the solution sequence also. This will help us improve the performance of original CX also.



- However, a single strategy cannot achieve the same performance of the combination these two strategies together.

The EA's Mechanism

(HLCX)

1. Remove depot (0) from parents

0	1	6	4	0	2	3	7	0	10	9	8	5	0

0	2	4	8	0	10	7	3	0	9	5	6	1	0

1	6	4	2	3	7	10	9	8	5	Parent 1

2	4	8	10	7	3	9	5	6	1	Parent 2

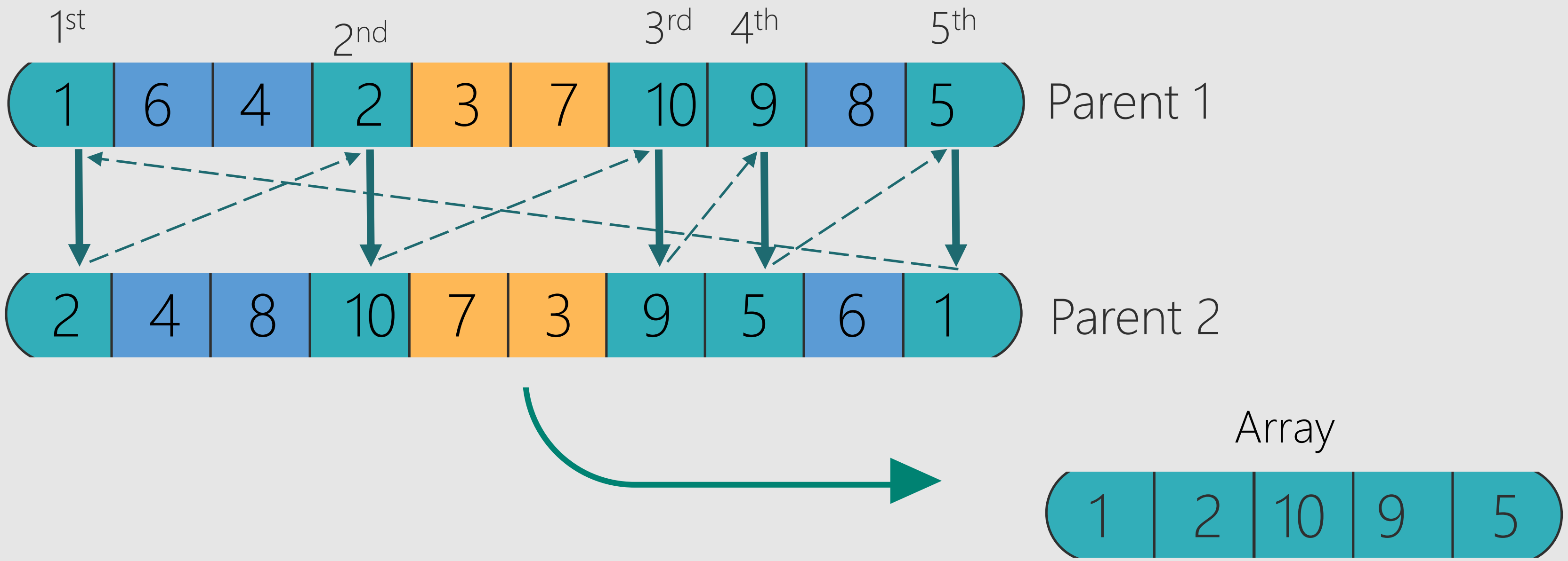
Initialization
- Encoding
- Decoding

Reproduction
- Duplicate Removal
- Tournament Selection
- Crossover
- Local Refinement

Selection
- selection
- Mutation
- 2-opt

The EA's Mechanism (HLCX)

2. Assign all customers to the cycles, and copy the longest cycle to the array



More Detail

The EA's Mechanism (HLCX)

3. Copy the all customers from one parent to the offspring, **except the customers in the array.**

Parent 2	2	4	8	10	7	3	9	5	6	1
Offspring	-	4	8	-	7	3	-	-	6	-

Array

1	2	10	9	5

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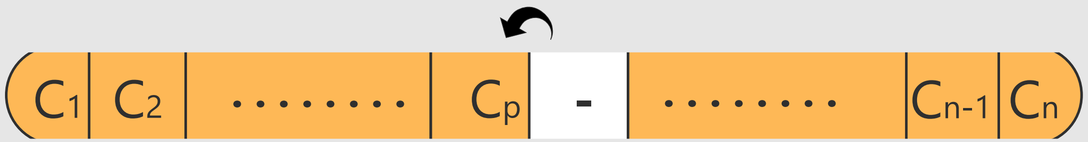
The EA's Mechanism (HLCX)

4. Fill in the leftmost empty position by choosing from the customer in array **which is closest to the previous customer (in term of distance).**



Offspring sequence

Array

$C_p \rightarrow X = 50$

*** $C_p \rightarrow Y = 40$**

$C_p \rightarrow Z = 75$

Offspring sequence

** Compare the 1st position with the next position since it has no previous position

Offspring sequence

Initialization
- Encoding
- Decoding

Reproduction
- Duplicate Removal
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- Crossover
- Local Refinement

Selection
- selection
- Mutation
- 2-opt

The EA's Mechanism (HLCX)

4. Fill in the leftmost empty position by choosing from the customer in array **which is closest to the previous customer (in term of distance).**

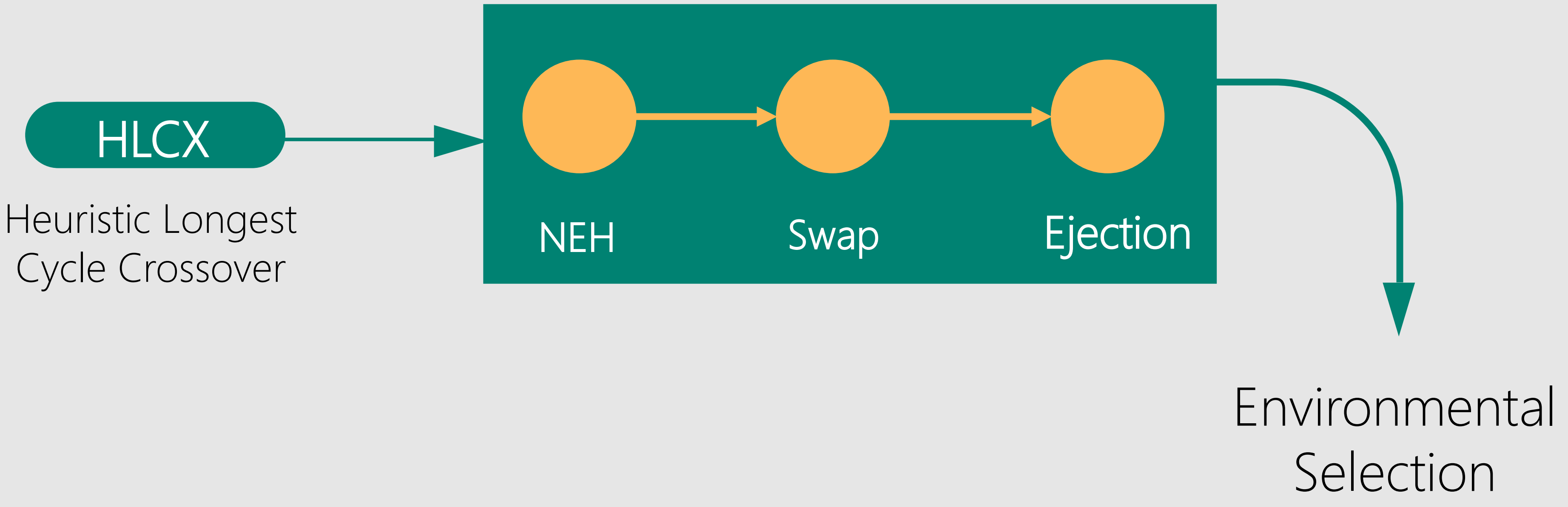
-	4	8	-	7	3	-	-	6	-	Offspring
10	4	8	-	7	3	-	-	6	-	Offspring
10	4	8	9	7	3	-	-	6	-	Offspring
10	4	8	9	7	3	1	5	6	2	Offspring (Done)

|1→4|	|2→4|	|10→4|	|9→4|	|5→4|
50 km.	32km.	20km.	25km.	45km.

|8→1|	|8→2|	|8→9|	|8→5|
78 km.	40km.	32km.	60km.

- The EA's Mechanism

(Local refinement)



Initialization
- Encoding
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Reproduction
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- Tournament Selection
- Crossover
- Local Refinement

Selection
- selection
- Mutation
- 2-opt

The EA's Mechanism (Local refinement)

- The local refinement is consisted of 3 operators including **NEH**, **Swap** and **Ejection**

NEH: Reinsert the random customer to the best position which provides the minimal distance (**must not violate maximum capacity**)

Current distance 66 Km.

4 Random customer

Current distance 50 Km.

The EA's Mechanism (Local refinement)

Swap: Swap a random customer with the customers for the other route which provides the minimal distance (**must not violate maximum capacity**)

0	1	6	0	2	3	0	4	5	7	0

Current distance 66 Km.

2 ⇆ 7 = 42 km

7 Random customer

X 3 (Maximum capacity : 100 Kg)

0	1	6	0	7	3	0	4	5	2	0

Current distance 50 Km.

Initialization
- Encoding
- Decoding

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- selection
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The EA's Mechanism (Local refinement)

Ejection (Apply with the top 10% largest demand customer)

0	1	6	0	2	3	0	4	5	7	0

3	6	4	2	1	7	5
70	65	55	25	20	10	15

3 Random customer ⇆ Random route :

0	4	5	7	0

{4,5} ∈ Set B

3 = Set B

55 + 15 = 70

0	1	6	0	4	2	5	0	3	7	0

3 ⇆ {4,5}

The EA's Mechanism

- Environmental selection
  - The best 100 solutions of each iteration are selected to be the candidate solution of the next iteration
- Mutation
  - Except the best 30 solutions, 10% of the solutions in the population are selected randomly to apply the swap operator, which exchanges two random customers without violating the capacity constraint to maintains population diversity
- 2-opt (with the best-found solution)
  - Apply 2-opt only to the best solution in the population at the last iteration, which helps us to remove the crosses in the route and reduce the travel distance.

Parameter Setting

- Population size = 100
- Generation number (iteration) = 100
- Crossover = 100%
- Local Refinement operators = 100%
- Mutation = 10%

Problem Instance

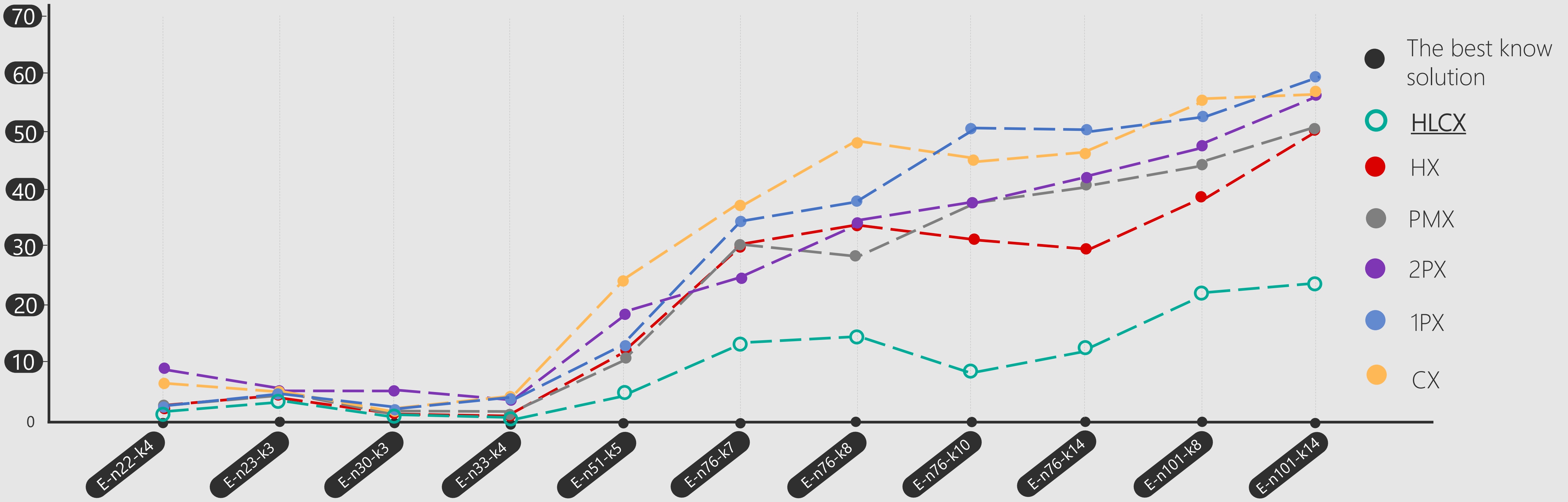
CVRPLIB
Capacitated vehicle Problem Library

http://vrp.atd-lab.inf.puc-rio.br/index.php/en/

- Set E instance problem (21 - 101 customers with 4 – 14 vehicles)

Crossover-only EA

Gap(%) between the best-found solutions be each crossover and the best-known solutions



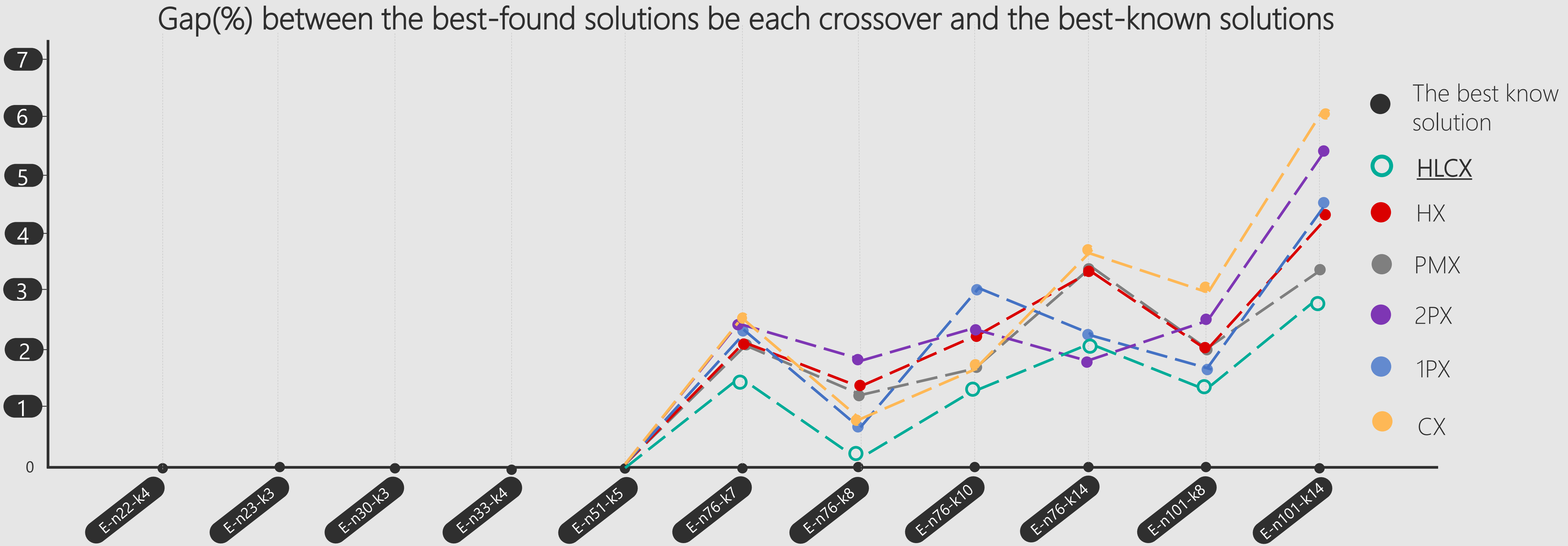
$$\text{Gap}(\%) = \frac{(\text{Min} - \text{BKS}) \times 100\%}{\text{BKS}}$$

Crossover-only EA

Performance comparison of six crossover-only EA

	BKS	HLCX			HX			PMX			CX			1PX			2PX		
		Min	%Gap	Avg	Min	%Gap	Avg	Min	%Gap	Avg	Min	%Gap	Avg	Min	%Gap	Avg	Min	%Gap	Avg
E-N22-K4	375	384	2.4	389.8	**381**	1.6	**381.0**	384	2.4	398.3	400	6.7	417.1	384	2.4	413.4	409	9.1	429.5
E-N23-K3	569	**594**	4.4	**597.1**	**594**	4.4	597.6	596	4.7	597.9	596	4.7	597.5	594	4.4	600	596	4.7	599.8
E-N30-K3	534	**539**	0.9	**543.1**	539	0.9	544.4	542	1.5	558.0	546	2.2	565.7	542	1.5	558.4	560	4.9	582.9
E-N33-K4	835	**836**	0.1	**844.5**	842	0.8	850.5	844	1.1	872.5	872	4.4	902.6	865	3.6	899.4	862	3.2	904.5
E-N51-K5	521	**549**	5.4	**573.8**	587	12.7	602.0	578	10.9	619.8	647	24.2	669.8	588	12.9	660.4	616	18.2	643.9
E-N76-K7	682	**776**	13.8	**816.4**	886	29.9	900.6	888	30.2	938.3	936	37.2	999.2	918	34.6	995.5	851	24.8	948.3
E-N76-K8	735	**846**	15.1	**881.7**	980	33.3	999.6	944	28.4	1012.5	1084	47.5	1142.1	1013	37.8	1079	986	34.1	1061.0
E-N76-K10	830	**906**	9.2	**985.9**	1088	31.1	1130.4	1142	37.6	1195.5	1202	44.8	1278.0	1246	50.1	1281	1142	37.6	1254.8
E-N76-K14	1021	**1154**	13.0	**1219.0**	1325	29.8	1400.3	1434	40.5	1551.2	1495	46.4	1609.4	1528	49.7	1597	1451	42.1	1541.0
E-N101-K8	815	**1001**	22.8	**1090.9**	1132	38.9	1164.3	1173	43.9	1263.4	1266	55.3	1327.0	1241	52.3	1296	1197	46.9	1296.2
E-N101-K14	1067	**1322**	23.9	**1437.9**	1600	50.0	1625.0	1593	49.3	1709.4	1671	56.6	1774.1	1696	59.0	1833	1669	56.4	1765.3

Complete EA



$$\text{Gap}(\%) = \frac{(\text{Min} - \text{BKS}) \times 100\%}{\text{BKS}}$$

Complete EA

Performance comparison of six complete EA

	BKS	HLCX			HX			PMX			CX			1PX			2PX		
		Min	%Gap	Avg	Min	%Gap	Avg	Min	%Gap	Avg	Min	%Gap	Avg	Min	%Gap	Avg	Min	%Gap	Avg
E-N22-K4	375	**375**	0	**375.0**	**375**	0	**375.0**	**375**	0	**375.0**	**375**	0	375.7	**375**	0	**375.0**	**375**	0	**375.0**
E-N23-K3	569	**569**	0	**569.0**	**569**	0	**569.0**	**569**	0	**569.0**	**569**	0	**569.0**	**569**	0	**569.0**	**569**	0	**569.0**
E-N30-K3	534	**534**	0	541.6	**534**	0	535.5	**534**	0	**534.9**	**534**	0	536.4	**534**	0	535.5	**534**	0	536.6
E-N33-K4	835	**835**	0	**835.0**	**835**	0	836.7	**835**	0	835.4	**835**	0	835.8	**835**	0	835.4	**835**	0	**835.0**
E-N51-K5	521	**521**	0	**521.0**	**521**	0	526.6	**521**	0	523.4	**521**	0	523.6	**521**	0	525.2	**521**	0	523.7
E-N76-K7	682	**692**	1.5	**697.1**	696	2.1	703.2	696	2.1	705.6	699	2.5	702.3	698	2.3	702.3	699	2.5	707.5
E-N76-K8	735	**737**	0.3	**743.3**	745	1.4	750.2	744	1.2	751.4	741	0.8	753.7	740	0.7	752.9	748	1.8	751.3
E-N76-K10	830	**842**	1.4	**854.2**	848	2.2	860.4	844	1.7	859.5	845	1.8	860.7	855	3.0	865.7	849	2.3	862.2
E-N76-K14	1021	1043	2.2	**1053.2**	1055	3.3	1066.3	1055	3.3	1066.4	1059	3.7	1070.5	1043	2.2	1063.6	**1039**	1.8	1062.7
E-N101-K8	815	**827**	1.5	**837.0**	831	2.0	853.0	831	2.0	841.0	840	3.1	849.4	829	1.7	851.0	835	2.5	853.0
E-N101-K14	1067	**1098**	2.9	**1124.7**	1113	4.3	1134.4	1103	3.4	1153.1	1131	6.0	1157.6	1115	4.5	1148.4	1125	5.4	1134.4

Conclusion

- The proposed idea including Longest cycle selection and Nearest neighbor heuristic which is the knowledge based of the problem can help operator preform better.

- This research will continue with two directions:

  - first, we will keep improving our algorithm for solving multi-objective and large-scale CVRP instances.

  - second, we will investigate the performance of the proposed HLCX in solving other combinatorial optimization problems.

Thanks for your attention

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