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Projective three-level main effects designs robust to model uncertainty

BY PI-WEN TSAI

Department of Applied Statistics, The University of Reading, PO Box 240, Earley Gate, Reading RG6 6FN, U.K.

piwen_tsai@hotmail.com

STEVEN G. GILMOUR

School of Mathematical Sciences, Queen Mary and Westfield College, Mile End Road, London E1 4NS, U.K.

s.g.gilmour@qmw.ac.uk

AND ROGER MEAD

Department of Applied Statistics, The University of Reading, PO Box 240, Earley Gate, Reading RG6 6FN, U.K.

r.mead@rdg.ac.uk

SUMMARY

This paper is concerned with designing experiments under the assumption that an analysis strategy is used that considers interactions in addition to main effects. A criterion which averages an approximation to A_s-efficiency over lower-dimensional projections of the design is introduced to compare designs. A columnwise design procedure is used to construct three-level designs for six factors in 18 runs. The projection efficiencies of these designs are explored and compared with designs obtained from the L_{18} orthogonal array. Results are also given for designs with 14 and 17 runs.

Some key words: Fractional factorial design; Response surface methodology; Screening experiment; Taguchi methods.

1. INTRODUCTION

Screening designs are widely used in industrial experiments to find the few factors which have major effects. A common assumption, described as effect sparsity by Box & Meyer (1986), is that only a small number of effects are important relative to the rest. Traditionally, design and analysis of screening experiments has been restricted to main effects only by assuming that all interactions are small and therefore negligible. We go beyond this traditional approach by assuming that an analysis strategy will be used that considers interactions in addition to main effects for the analysis of screening experiments. The question of how to design experiments for this analysis strategy is therefore of interest.

We will use, as an example, an experiment described by Logothetis (1990), who presented a detailed analysis of experimental data from a plasma etching process. In the experiment, six design factors, labelled $F_1, \ldots, F_6$, each at three levels, were assigned to

the columns of an $L_{18}(3^6)$ array, which is design L18_1 in Table 2 below, in which $-$, 0 and $+$ refer to the low, middle and high levels of a factor. This $L_{18}(3^6)$ design array is obtained by omitting a column from an $L_{18}(3^7)$ orthogonal array which is recommended by Taguchi (1986) for three-level designs. It is an efficient design for estimating, at the expense of only 18 trials, the main effects of six three-level factors under the assumption that no interaction exists.

Motivated by the procedure of Hamada & Wu (1992), a two-stage analysis is followed that considers interactions in addition to main effects. First, the main effects model is fitted to identify the factors which have non-negligible main effects on the response, called the active factors by Box & Meyer (1986). In this experiment the linear effects of F_1, F_5 and F_6, and probably the quadratic effect of F_2 and the linear effect of F_4, are large; see Tsai, Gilmour & Mead (1996) for details. Secondly, a stepwise regression procedure is used for selecting a suitable fitted model, obeying functional marginality (McCullagh & Nelder, 1989, Ch. 3), called strong heredity by Chipman (1996), for the interpretation of the data. When we specify the significance level for adding or dropping a term in the stepwise regression model to be 0·1, the final model selected is a model which contains linear effects of F_1, F_2, F_5 and F_6 and the linear by linear interaction of F_1 and F_5. Logothetis (1990), Fearn (1992) and Tsai et al. (1996) discussed other aspects of the analysis of these data.

In the general problem, assume that only some of the design factors are active after the analysis of variance for the factors' main effects. Columns in the design array that correspond to the other, inert factors can be omitted. The combinations of levels for the set of active factors of the original design is a lower-dimensional projection of the design. A particular projective design obtained by omitting the inert factors does not always provide the best arrangement possible. For example, for the $L_{18}(3^6)$ array used in the experiment, there are only nine distinct points in a three-factor projective design for factors F_1, F_3 and F_4. Clearly, this is not an efficient design. Experimenters do not in advance know which are the important factors and what the final model will be. Hence, it is important that designs used for screening experiments can project on to good lower-dimensional designs for various sets of factors and thus provide reasonably efficient parameter estimates for a range of possible models. The projection properties of two-level designs have recently been studied by a number of authors; see Lin & Draper (1992), Lin (1993), Cheng (1995, 1998), Box & Tyssedal (1996), Chen (1998) and Aggarwal & Kaul (1999).

A new overall design criterion based on approximate variances is proposed in § 2 to evaluate the worth of designs over a range of possible models. A columnwise procedure for the construction of designs in various run sizes is described in § 3. In § 4 an example is presented of the construction of three-level designs for six factors in 18 runs and results are also given for 17 and 14 runs.

2. An overall design criterion based on approximate variance

In this section we develop a design criterion which is an approximation to the mean A_s-efficiency, ignoring the intercept, over all models that we will consider fitting. The idea of using average A-efficiency over several models was discussed by Wu (1993) in the context of two-level supersaturated designs. He considered averaging A over all main effects models with a small number of factors. Our approximation allows the averaging to be done over a very much larger number of models, which may include interactions as well as main effects.

Assume that the maximal model of interest, the full second-order polynomial in our case, is $E(y) = X\beta$, where y is an $N \times 1$ vector of observations, X is an $N \times (v+1)$ design matrix and $\beta = (\beta_0, \ldots, \beta_v)'$ is a $(v+1) \times 1$ vector of parameters. The information matrix for this model is $X'X$ and the variance-covariance matrix of the least squares estimator of β divided by σ^2 is $(X'X)^{-1}$. Let a_{ij}, for $i, j = 0, \ldots, v$, be the elements of $X'X$ and let c_{ii}, for $i = 0, \ldots, v$, be the diagonal elements of $(X'X)^{-1}$. Ideally, the off-diagonal elements of $X'X$ are small compared with the diagonal elements of the matrix, that is $a_{st} \ll a_{ss}$ and $a_{st} \ll a_{tt}$ for $s \neq t$. Assume that the higher-order terms in the diagonal expansion of the determinant of $X'X$, as in Hohn (1973, p. 303), are small and can be ignored. Then c_{ii} can be approximated as

$$c_{ii} \simeq \left(\prod_{\substack{s=0 \\ s \neq i}}^{v} a_{ss} \right) \left(1 - \sum_{\substack{s=0 \\ s \neq i}}^{v-1} \sum_{\substack{t=s+1 \\ t \neq i}}^{v} \frac{a_{st}^2}{a_{ss} a_{tt}} \right) \Bigg/ \left\{ \left(\prod_{s=0}^{v} a_{ss} \right) \left(1 - \sum_{s=0}^{v-1} \sum_{t=s+1}^{v} \frac{a_{st}^2}{a_{ss} a_{tt}} \right) \right\}. \tag{1}$$

Applying the generalised Taylor's theorem and ignoring higher-order terms, we approximate equation (1) as

$$c_{ii} \simeq \frac{1}{a_{ii}} \left(1 - \sum_{\substack{s=0 \\ s \neq i}}^{v-1} \sum_{\substack{t=s+1 \\ t \neq i}}^{v} \frac{a_{st}^2}{a_{ss} a_{tt}} \right) \left(1 + \sum_{s=0}^{v-1} \sum_{t=s+1}^{v} \frac{a_{st}^2}{a_{ss} a_{tt}} \right).$$

Again ignoring higher-order terms, we have

$$c_{ii} \simeq \frac{1}{a_{ii}} \left(1 + \sum_{s=0}^{v-1} \sum_{t=s+1}^{v} \frac{a_{st}^2}{a_{ss} a_{tt}} - \sum_{\substack{s=0 \\ s \neq i}}^{v-1} \sum_{\substack{t=s+1 \\ t \neq i}}^{v} \frac{a_{st}^2}{a_{ss} a_{tt}} \right) = \sum_{j=0}^{v} \frac{1}{a_{ii}} \frac{a_{ij}^2}{a_{ii} a_{jj}}.$$

We further define a $(v+1) \times (v+1)$ non-symmetric matrix R whose elements, r_{ij}, for $i, j = 0, \ldots, v$, are defined as

$$r_{ij} = \frac{1}{a_{ii}} \frac{a_{ij}^2}{a_{ii} a_{jj}}.$$

Hence $\operatorname{var}(\hat{\beta}_i) \simeq \sum_{j=0}^{v} r_{ij}$. Note that $r_{ii} = 1/a_{ii}$ and in general $r_{ij} \neq r_{ji}$ for $i \neq j$.

We will end up fitting a model, M_s, which contains a subset of effects of the maximal model. Define

$$M_s(i, j) = \begin{cases} 1 & \text{if effects } i \text{ and } j \text{ are both in model } M_s, \\ 0 & \text{otherwise.} \end{cases}$$

Then the approximate variance for estimating parameter i in the model M_s can be written as

$$\operatorname{var}(\hat{\beta}_i \mid M_s) \simeq \sum_{j=0}^{v} r_{ij} M_s(i, j), \tag{2}$$

so that an element r_{ij} of R appears in (2) only if terms i and j are both in model M_s. The sum of the approximate variances for the effects in this model, excluding the intercept, is an approximation to the criterion of A_s-efficiency for this model and can be written as $\sum_{i=1}^{v} \sum_{j=0}^{v} r_{ij} M_s(i, j)$.

For an N-point design, models with more than N parameters are not estimable. Suppose that there are n_0 estimable models. An overall design criterion, that averages this approximation to the criterion of A_s-optimality over the n_0 models, is introduced to evaluate the worth of a design. This overall criterion, denoted by $Q(\Gamma^{(k)})$, is defined as

$$Q(\Gamma^{(k)}) = \frac{1}{n_0} \sum_{s=1}^{n_0} \sum_{i=1}^{v} \sum_{j=0}^{v} r_{ij} M_s(i, j). \tag{3}$$

This is used to explore the projection efficiencies of the design. Designs with lower $Q(\Gamma^{(k)})$ are more likely to have efficient projections and on average can provide better parameter estimation over a range of models than designs with higher $Q(\Gamma^{(k)})$.

Let w_{ij} for $i, j = 0, \ldots, v$ denote an element of a $(v+1) \times (v+1)$ matrix W such that $w_{ij} = \sum_{s=1}^{n_0} M_s(i, j)$, which is the number of models that contain both terms i and j. Note that $w_{ij} = w_{ji}$, for $i \neq j$. Then the design criterion given in equation (3) can be written as

$$Q(\Gamma^{(k)}) = \frac{1}{n_0} \sum_{i=1}^{v} \sum_{j=0}^{v} r_{ij} w_{ij}.$$

This is the sum of the elements of R, excluding row 0, weighted by the numbers of possible models containing the effects to which these elements refer and divided by the number of models of interest, where r_{ij} in R depends on the design but w_{ij} is the same for each design with the same N.

This design criterion does not require the inversion of the information matrices for the models of interest. It is simple and computationally inexpensive, and therefore it can quickly provide us with information on the worth of a design over a wide range of models. It allows us to search over a large number of possible designs, whereas algorithms based on other criteria which work with the inverse of a matrix can only look at relatively few designs. Note that, if the diagonal elements of $X'X$ are not relatively small compared with the off-diagonal elements, then the approximate variances will not be very accurate. However, for the purpose of comparing designs, designs with large values of the off-diagonal elements of $X'X$ tend to have large values of $Q(\Gamma^{(k)})$. This criterion remains useful for selecting efficient designs.

3. A COLUMNWISE DESIGN PROCEDURE

To construct a design with good projective properties, a procedure is used which forces the low-dimensional projections to have the desired properties. We start with lower-dimensional designs and add other factors one by one to construct designs for more factors. A computer search routine is used to ensure that designs that are generated can always project on to designs for two factors in which the pair of factors represented by the columns of the design are nearly orthogonal to the intercept and to each other. This design procedure uses $Q(\Gamma^{(k)})$ to compare designs. Columnwise algorithms have previously been used for the construction of saturated and supersaturated designs; see Lin (1993), Wu (1993) and Li & Wu (1997).

In the columnwise design procedure, we restrict our attention to designs in which each level of a factor appears as nearly as possible equally often, and each combination of the levels of each pair of factors appears as nearly as possible equally often. The former is referred to as the level-balance criterion, and the latter as the orthogonality criterion. The procedure is illustrated with the example of six factors in 18 runs.

4. Examples of constructing designs

4·1. *Six factors in* 18 *runs*

Consider an 18-run design for six factors in which each factor has three levels. In the two-stage analysis procedure, the main effects model is used to identify the active factors. For $N = 18$, it is important to generate designs that can always project on to designs for two factors in which the pair of factors represented by the columns of the possible design are orthogonal to the intercept and to each other. Thus, designs that are generated are equally efficient for the six-factor main effects model and for any submodel of this model. According to the level-balance and orthogonality criteria, each column must have each factor level six times and each pair of columns must have each combination twice. Thus the columns of the two-factor design, denoted by $\Gamma^{(2)} = (g_1, g_2)$, are

$$g_1 = [-, -, -, -, -, -, 0, 0, 0, 0, 0, 0, +, +, +, +, +, +]',$$
$$g_2 = [-, -, 0, 0, +, +, -, -, 0, 0, +, +, -, -, 0, 0, +, +]'.$$

The estimated main effects of these two factors are orthogonal to the intercept and to each other. The design criterion for this two-factor design, $Q(\Gamma^{(2)})$, has value 0·2731, which is therefore the same as the average A_s-efficiency for the 12 possible models.

This two-factor design is used as a starting design for the construction of designs for more factors. For the construction of designs for three factors, we generate a set of columns which satisfy the level-balance criterion and whose combinations with g_1 in $\Gamma^{(2)}$ satisfy the orthogonality criterion. There are $\{6!/(2!2!2!)\}^3 = 729\,000$ such columns. A computer search routine is then used to check whether or not the combinations of the levels of the third column and g_2 satisfy the orthogonality criterion. If they do, the new column is kept; otherwise, it is discarded.

Designs which can be obtained from each other by relabelling some of the factors, or by switching all plus and minus signs of some of the factors, are said to be from the same design family. Only one design in a design family is kept, since the other members of the family have the same properties. If designs $\Gamma_i^{(k)}$ and $\Gamma_j^{(k)}$ are in the same design family, then the elements in $X'X$ for the k-factor second-order model of $\Gamma_i^{(k)}$ and $\Gamma_j^{(k)}$ are the same in magnitude, and thus $Q(\Gamma_i^{(k)})$ and $Q(\Gamma_j^{(k)})$ will be the same. Here, we use $Q(\Gamma^{(k)})$ to determine whether or not designs are from the same design family. Although it is possible that some designs will be missed, there is no certain way to avoid this without looking at every possible design, when the problem soon becomes unmanageable.

When we apply the search routine, 23 436 out of the possible columns are kept. Among them there are 13 different design families, other designs being obtained by relabelling the factors or by switching all signs of the factors.

For each of these designs for three factors, the linear and quadratic effects of a factor are orthogonal to those of other factors and are orthogonal to the linear by linear interactions involving that factor. These designs permit the main effects of the factors to be estimated independently assuming no interaction exists. However, in these designs the linear and quadratic effects of a factor are sometimes correlated with the interaction not involving that factor, and the interactions are correlated with each other. For a three-factor design, there are a total of 94 possible models that we consider. Table 1 gives values of $Q(\Gamma^{(3)})$ and the average of the A_s-criterion of a design over the possible models, denoted by $A_{\text{all}}(\Gamma^{(3)})$, for these designs. It shows that the differences between $Q(\Gamma^{(3)})$ and $A_{\text{all}}(\Gamma^{(3)})$ for these designs are small, especially for the top five designs where the nonorthogonalities between effects in the designs are small. This confirms that $Q(\Gamma^{(3)})$ is a reasonable approxi-

mation for $A_{\text{all}}(\Gamma^{(3)})$ and, more importantly, that designs with lower $Q(\Gamma^{(3)})$ on average provide better parameter estimates over all possible models than those with higher $Q(\Gamma^{(3)})$. Note that designs 12 and 13 are sets of just nine duplicated points, so there are not enough experimental runs to obtain unique estimates for all the effects of the three-factor second-order model. The values for $A_{\text{all}}(\Gamma^{(3)})$ of these two designs are not defined.

Table 1. *Properties of the* 13 *designs for three factors in* 18 *runs*

Design	$Q(\Gamma^{(3)})$	$A_{\text{all}}(\Gamma^{(3)})$	Design	$Q(\Gamma^{(3)})$	$A_{\text{all}}(\Gamma^{(3)})$
1	0·5148	0·5157	8	0·5635	0·6063
2	0·5236	0·5259	9	0·5656	0·5960
3	0·5301	0·5389	10	0·5683	0·5950
4	0·5304	0·5392	11	0·5945	0·6663
5	0·5328	0·5367	12	0·5945	∞
6	0·5378	0·5615	13	0·5945	∞
7	0·5426	0·5497			

Continuing to add factors in the same way, we find 129 four-factor designs, 320 five-factor designs and 440 six-factor designs. In each of these six-factor designs, the levels of each factor appear exactly six times and the nine level-combinations for all 15 pairs of factors appear exactly twice. However, the combinations for three, four, five and six factors and the nonorthogonalities between the main effects and the interactions are different.

Table 2 gives the designs for six factors in 18 runs with the six lowest values of $Q(\Gamma^{(6)})$. Additionally, it gives the plans for two six-factor designs in 18 runs, named L18_1 and L18_2, which are obtained by omitting a column from an $L_{18}(3^7)$ orthogonal array. Note that L18_1 and L18_2 are also among the 440 six-factor designs that are generated by our design procedure. In terms of $Q(\Gamma^{(6)})$, they rank 339 and 342 respectively.

These designs are equally efficient for fitting the six-factor main effects model and any sub-model of it. Table 3 summarises the projective properties for these six-factor designs in 18 runs, where Ave $Q(\Gamma^{(f)})$ averages the overall design criteria of the projective designs of f factors, $f = 5, 4, 3$. It shows that designs that are generated by the design procedure with low $Q(\Gamma^{(6)})$ have better projective properties for various values of f than L18_1 and L18_2. These designs on average can provide better parameter estimates for a wide range of possible models.

The projection efficiencies of designs over lower dimensions are important for the construction of designs for p factors with low $Q(\Gamma^{(p)})$. To save computational time, it is possible to use only the $(p-1)$-factor designs whose overall design criteria are within about 5% of that for the best design for which there is also at least one additional column whose combination with the $(p-1)$ columns of the design satisfies the orthogonality criterion. This was done for the examples in the rest of this section.

4·2. *Designs with fewer runs*

The procedure defined and used in § 4·1 to obtain designs for six factors in 18 runs can be used for any number of factors in any number of runs and is not, for example, restricted to multiples of nine or three runs. Thus, if there are insufficient resources to perform 18 runs, six factors could be studied in fewer runs. We have studied six factors in 13 to 17 runs and here we give a brief summary of results in two of the more interesting cases.

Table 2. *Plans for some designs for six factors in* 18 *runs*

Design 1	Design 2	Design 3	Design 4
− − − − − 0	− − − − 0 −	− − − − − 0	− − − − 0 −
− − 0 + + −	− − 0 + − 0	− − 0 + 0 −	− − 0 0 − +
− 0 − 0 0 −	− 0 − 0 − +	− 0 − 0 + −	− 0 − + + +
− 0 + − + +	− 0 + − + 0	− 0 + − 0 +	− 0 + + − 0
− + 0 0 − +	− + 0 0 + −	− + 0 0 − +	− + 0 − + 0
− + + + 0 0	− + + + 0 +	− + + + + 0	− + + 0 0 −
0 − + − 0 −	0 − 0 0 + +	0 − 0 0 0 0	0 − + − + 0
0 − + 0 − +	0 − + − − +	0 − + − + −	0 − + + 0 +
0 0 − + + +	0 0 − + + 0	0 0 − + 0 +	0 0 − 0 + −
0 0 0 0 0 0	0 0 + 0 0 −	0 0 + 0 − 0	0 0 0 0 0 0
0 + − + − −	0 + − + − −	0 + − + − −	0 + − − − +
0 + 0 − + 0	0 + 0 − 0 0	0 + 0 − + +	0 + 0 + − −
+ − − 0 + 0	+ − − 0 0 0	+ − − 0 + +	+ − − 0 − 0
+ − 0 + 0 +	+ − + + + −	+ − + + − +	+ − 0 + + −
+ 0 0 − − −	+ 0 0 − − −	+ 0 0 − − −	+ 0 0 − 0 +
+ 0 + + − 0	+ 0 0 + 0 +	+ 0 0 + + 0	+ 0 + − − −
+ + − − 0 +	+ + − − + +	+ + − − 0 0	+ + − + 0 0
+ + + 0 + −	+ + + 0 − 0	+ + + 0 0 −	+ + + 0 + +

Design 5	Design 6	L18_1	L18_2
− − − − 0 0	− − − − 0 0	− − − − − −	− − − − − −
− − 0 0 − −	− − 0 + 0 −	− − + + 0 0	− − 0 0 + +
− 0 − + + +	− 0 − 0 − +	− 0 − − + +	− 0 − + 0 +
− 0 + + − 0	− 0 + − + −	− 0 0 0 0 0	− 0 + − + 0
− + 0 − + −	− + 0 0 + +	− + 0 0 − −	− + 0 + − 0
− + + 0 0 +	− + + + − 0	− + + + + +	− + + 0 0 −
0 − + − + +	0 − 0 0 + 0	0 − − 0 0 +	0 − − + + 0
0 − + + 0 −	0 − + − − +	0 − 0 + − +	0 − + − 0 +
0 0 − − − −	0 0 − + + −	0 0 0 + + −	0 0 0 0 0 0
0 0 0 0 0 0	0 0 + 0 0 0	0 0 + − 0 −	0 0 + + − −
0 + − 0 + 0	0 + − + 0 +	0 + − 0 + 0	0 + − 0 − +
0 + 0 + − +	0 + 0 − − −	0 + + − − 0	0 + 0 − + −
+ − − 0 − +	+ − − 0 − −	+ − 0 − + 0	+ − 0 + 0 −
+ − 0 + + 0	+ − + + + +	+ − + 0 + −	+ − + 0 − 0
+ 0 0 − 0 +	+ 0 0 − 0 +	+ 0 − + − 0	+ 0 − 0 + −
+ 0 + 0 + −	+ 0 0 + − 0	+ 0 + 0 − +	+ 0 0 − − +
+ + − + 0 −	+ + − − + 0	+ + − + 0 −	+ + − − 0 0
+ + + − − 0	+ + + 0 0 −	+ + 0 − 0 +	+ + + + + +

Case 1: 17 *runs.* Lin (1993) found that the best two-level designs in $4n-1$ runs could be obtained by deleting any point from the best designs in $4n$ runs. Similarly we found that the best 17-run designs were obtained by deleting a point from the best 18-run designs. However, with three-level factors it is important to delete the appropriate point. It turns out to be better to delete points with as many 0's as possible, rather than points with many ± 1's. The best three designs, with $Q(\Gamma^{(6)})$ equal to 2·1923, 2·2065 and 2·2094 respectively, are obtained by deleting the point (0, 0, 0, 0, 0, 0) from Design 1 in Table 2, by deleting the point (0, −, 0, 0, 0, 0) from Design 3 in Table 2 and by deleting the point (0, 0, 0, 0, 0, 0) from Design 4 in Table 2. Note that Design 2 in Table 2 contains no point with more than four 0's.

Table 3. *Summary of properties of the designs in Table 2*

Design	$Q(\Gamma^{(6)})$	Ave $Q(\Gamma^{(f)})$ $f=5$	$f=4$	$f=3$
1	2·2656	1·6362	0.9726	0·5326
2	2·2692	1·6294	0·9650	0·5300
3	2·2717	1·6388	0·9731	0·5326
4	2·2871	1·6466	0·9739	0·5324
5	2·2875	1·6427	0·9749	0·5331
6	2·2891	1·6474	0·9740	0·5324
L18_1	2·4515	1·7341	1·0080	0·5400
L18_2	2·4524	1·7055	0·9853	0·5328

Table 4. *Properties of designs for six factors in 14 runs*

Design	$Q(\Gamma^{(6)})$	A_s
1	1·9744	0·1478
2	2·0365	0·1582
3	2·0617	0·1643
4	2·0738	0·1668
A1	2·8860	0·1384
A2	2·8910	0·1384

Table 5. *Plans for designs for six factors in 14 runs*

Design 1	Design 2	Design A1	Design A2
− − − − − 0	− − − − − 0	− − − 0 0 0	− − + + + +
− − + + 0 −	− − + + 0 −	− 0 0 + + −	− 0 − − − −
− 0 − + + +	− 0 − + + +	− + + − − +	− + 0 0 0 0
− + 0 − + −	− + 0 − + −	0 − − − + −	0 − − − 0 0
− + + 0 − +	− + + 0 − +	0 − 0 + 0 +	0 0 0 + 0 −
0 − + − + +	0 − + − + +	0 0 − + − 0	0 0 0 + + +
0 0 0 0 0 0	0 0 0 0 0 −	0 0 + − 0 +	0 0 + 0 − 0
0 0 0 0 0 0	0 0 0 0 0 0	0 0 + 0 0 −	0 + − 0 + −
0 + − + − −	0 + − + − −	0 + 0 0 + 0	0 + 0 − − +
+ − − 0 + −	+ − − 0 + −	+ − 0 0 − −	+ − 0 0 − −
+ − 0 + − +	+ − 0 + − +	+ − + + + 0	+ 0 − 0 0 +
+ 0 + − − −	+ 0 + − − 0	+ 0 − 0 + +	+ 0 0 − + 0
+ + − − 0 +	+ + − − 0 +	+ 0 0 − 0 0	+ + − + − 0
+ + + + + 0	+ + + + + 0	+ + − + 0 −	+ + + − 0 −

Case 2: 14 *runs.* The values of $Q(\Gamma^{(6)})$ and of A_s for the best four designs are shown in Table 4, along with the corresponding values for two A-optimal designs for the main effects model obtained from the SAS procedure OPTEX (SAS Institute, 1995, Pt. 6). The designs produced by our procedure sacrifice a little in terms of the main effects model in order to gain a lot over all possible models which might be fitted. Unusually, in this case Design 1 appears clearly better than the other designs obtained. Designs 1, 2, A1 and A2 are shown in Table 5. Design 1 consists of six pairs of foldover points, each with one factor at 0, plus two centre points. Note also that, unlike the designs we obtained, the A-optimal designs do not satisfy the level-balance criterion.

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