Three Kinds of Butterfly Effects Within Lorenz Models

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Abstract

This talk presents three kinds of butterfly effects, which were defined based on new insights on the characteristics of classical Lorenz models and a generalized Lorenz model. The butterfly effect of the first kind (BE1) refers to the sensitive dependence of solutions on initial conditions, while the butterfly effect of the second kind (BE2) relates to the hypothetical role of tiny initial perturbations in creating an organized large-scale system at distant locations. The butterfly effect of the third kind (BE3), also known as the real butterfly effect, involves the contribution of small-scale processes to the finite predictability of large-scale processes in highly turbulent regions with a (-5/3) kinetic energy spectrum. These three kinds of butterfly effects differ from each other, with BE1 being chaos, BE2 being a metaphorical analogy, and BE3 having a brief history. Major features of boundedness and recurrence within BE1 indicate finite errors (i.e., no blowup solutions), and coexisting chaotic and nonchaotic attractors suggest that BE1 may or may not appear, yielding distinct predictability. Finally, a popular but inaccurate analogy for butterfly effect and chaos, as listed by the following folklore, will be discussed.

> "For want of a nail, the shoe was lost.
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> For want of a shoe, the horse was lost.
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> For want of a horse, the rider was lost.
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> For want of a rider, the battle was lost.
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> For want of a battle, the kingdom was lost.
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> And all for the want of a horseshoe nail."

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