

4-2 小考 1 參考解法

1.

$$\bar{x} = \frac{2+6+3+4}{4} = \frac{15}{4}, \quad \bar{y} = \frac{3+8+6+10}{4} = \frac{27}{4}$$

$$x_i - \bar{x} : \frac{-7}{4}, \frac{9}{4}, \frac{-3}{4}, \frac{1}{4}$$

$$y_i - \bar{y} : \frac{-15}{4}, \frac{5}{4}, \frac{-3}{4}, \frac{13}{4}$$

$$S_{XY} = \left(\frac{-7}{4}\right)\left(\frac{-15}{4}\right) + \frac{9}{4} \times \frac{5}{4} + \left(\frac{-3}{4}\right)\left(\frac{-3}{4}\right) + \frac{1}{4} \times \frac{13}{4} = \frac{105+45+9+13}{16} = \frac{172}{16}$$

$$S_{XX} = \left(\frac{-7}{4}\right)^2 + \left(\frac{9}{4}\right)^2 + \left(\frac{-3}{4}\right)^2 + \left(\frac{1}{4}\right)^2 = \frac{49+81+9+1}{16} = \frac{140}{16}$$

$$b = \frac{S_{XY}}{S_{XX}} = \frac{172/16}{140/16} = \frac{172}{140} = \frac{43}{35}$$

所以迴歸直線： $y = a + \frac{43}{35}x$

因為 $(\bar{x}, \bar{y}) = \left(\frac{15}{4}, \frac{27}{4}\right)$ 是迴歸直線上的一點，所以代入迴歸直線求 a

$$\frac{27}{4} = a + \frac{43}{35} \times \frac{15}{4} \Rightarrow a = \frac{27}{4} - \frac{43 \times 15}{140} = \frac{945 - 645}{140} = \frac{300}{140} = \frac{15}{7}$$

所以迴歸直線： $y = \frac{15}{7} + \frac{43}{35}x$ #

2.

$$\text{相關係數} = r = \frac{S_{XY}}{\sqrt{S_{XX}} \sqrt{S_{YY}}}$$

已知 $r = 0$ ，所以 $\frac{S_{XY}}{\sqrt{S_{XX}} \sqrt{S_{YY}}} = 0 \Rightarrow S_{XY} = 0$

$$\bar{x} = \frac{4+1+3+2}{4} = \frac{10}{4}, \quad \bar{y} = \frac{3+k+2+5}{4} = \frac{10+k}{4}$$

$$S_{XY} = 4 \times 3 + 1 \times k + 3 \times 2 + 2 \times 5 - 4 \times \frac{10}{4} \times \frac{10+k}{4} = 12 + k + 6 + 10 - \frac{5(10+k)}{2} = 28 + k - \frac{5(10+k)}{2}$$

因為 $S_{XY} = 0$ ，所以 $28 + k - \frac{5(10+k)}{2} = 0 \Rightarrow 2(28+k) - 5(10+k) = 0 \Rightarrow 6 - 3k = 0 \Rightarrow k = 2$ #

3. (1)(2)參考習作的 2.4 題練習看看. #

$$(1) \text{相關係數} = \frac{\sqrt{10}}{8} \left(\frac{5}{4\sqrt{10}} \right)$$

$$(2) \text{迴歸直線} : y = 40 + 0.5x$$

4.

$$(1) \bar{x} = \frac{1}{10} \sum_{i=1}^{10} x_i = \frac{15}{10} = 1.5, \quad \bar{y} = \frac{1}{10} \sum_{i=1}^{10} y_i = \frac{18}{10} = 1.8 \quad \#$$

$$(2) \sigma_X = \sqrt{\frac{1}{10} \left(\sum_{i=1}^{10} x_i^2 - 10\bar{x}^2 \right)} = \sqrt{\frac{1}{10} (55 - 10 \times 1.5^2)} = \sqrt{\frac{55 - 22.5}{10}} = \sqrt{3.25} = \frac{\sqrt{13}}{2}$$

$$\sigma_Y = \sqrt{\frac{1}{10} \left(\sum_{i=1}^{10} y_i^2 - 10\bar{y}^2 \right)} = \sqrt{\frac{1}{10} (50 - 10 \times 1.8^2)} = \sqrt{\frac{50 - 32.4}{10}} = \sqrt{1.76} = \frac{2\sqrt{11}}{5} \quad \#$$

$$(3) S_{XY} = \sum_{i=1}^{10} x_i y_i - 10\bar{x}\bar{y} = 53 - 10 \times 1.5 \times 1.8 = 53 - 27 = 26$$

$$r = \frac{S_{XY}}{n\sigma_X\sigma_Y} = \frac{26}{10 \times \frac{\sqrt{13}}{2} \times \frac{2\sqrt{11}}{5}} = \frac{13}{\sqrt{143}} = \frac{\sqrt{143}}{11} \quad \#$$

$$(4) S_{XX} = \sum_{i=1}^{10} x_i^2 - 10\bar{x}^2 = 55 - 10 \times 1.5^2 = 55 - 22.5 = 32.5$$

$$b = \frac{S_{XY}}{S_{XX}} = \frac{26}{32.5} = \frac{260}{325} = 0.8$$

所以迴歸直線： $y = a + 0.8x$

因為 $(\bar{x}, \bar{y}) = (1.5, 1.8)$ 是迴歸直線上的一點，所以代入迴歸直線求 a

$$1.8 = a + 0.8 \times 1.5 \Rightarrow a = 1.8 - 1.2 = 0.6$$

所以迴歸直線： $y = 0.6 + 0.8x$ #

5.

(1)

$$\bar{x} = \frac{40+45+50+55+60}{5} = 50, \quad \bar{y} = \frac{55+51+48+41+35}{5} = 46$$

$$x_i - \bar{x} : -10, -5, 0, 5, 10$$

$$y_i - \bar{y} : 9, 5, 2, -5, -11$$

$$\sigma_X = \sqrt{\frac{(-10)^2 + (-5)^2 + 0^2 + 5^2 + 10^2}{5}} = \sqrt{50}$$

$$\sigma_Y = \sqrt{\frac{9^2 + 5^2 + 2^2 + (-5)^2 + (-11)^2}{5}} = \sqrt{\frac{256}{5}}$$

標準化太麻煩，所以思考有沒有別的作法。

標準化後的相關係數與原本的相關係數相同，所以我們只要算原本的相關係數即可。

Ps：直接標準化去算也是可以的喔！

$$S_{XY} = (-10) \times 9 + (-5) \times 5 + 0 \times 2 + 5 \times (-5) + 10 \times (-11) = -90 - 25 - 25 - 110 = -250$$

$$S_{XX} = (-10)^2 + (-5)^2 + 0^2 + 5^2 + 10^2 = 100 + 25 + 25 + 100 = 250$$

$$S_{YY} = 9^2 + 5^2 + 2^2 + (-5)^2 + (-11)^2 = 81 + 25 + 4 + 25 + 121 = 256$$

$$\text{相關係數} = r = \frac{S_{XY}}{\sqrt{S_{XX}} \sqrt{S_{YY}}} = \frac{-250}{\sqrt{250} \sqrt{256}} = \frac{-250}{5\sqrt{10} \times 16} = \frac{-25}{8\sqrt{10}} \#$$

(2)

※標準數據的迴歸直線： $y = rx$

※標準化之後的數據是一組標準數據。

$$\text{所以迴歸直線：} y = \frac{-25}{8\sqrt{10}} x \#$$