

4-1 小考 1 參考解法

1.

$$(1) \text{算術平均數} = \bar{x} = \frac{175+165+165+195+185+165+195}{8} = 180 \quad \#$$

(2) 將數字由小排到大：165、165、165、175、185、195、195、195

$$\text{中位數} = \frac{175+185}{2} = 180 \quad \#$$

(3) 眾數 = 165 和 195 #

$$(4) \text{標準差} = \sigma = \sqrt{\frac{(-5)^2 + (-15)^2 + (-15)^2 + (-5)^2 + 5^2 + 15^2 + 15^2 + 15^2}{8}} = \sqrt{175} = 5\sqrt{7} \doteq 13.229 \quad \#$$

$$2. \quad \text{加權平均數} = \frac{80 \times 5 + 76 \times 5 + 95 \times 6 + 85 \times 4 + 76 \times 4 + 80 \times 2}{26} = \frac{2154}{26} \doteq 52.85 \quad \#$$

3.

$$(1) \text{平均身高} = \frac{165 \times 1 + 175 \times 15 + 185 \times 15 + 195 \times 1}{32} = 180 \text{公分} \quad \#$$

$$(2) \text{變異數} = \sigma^2 = \frac{(-15)^2 \times 1 + (-5)^2 \times 15 + 5^2 \times 15 + 15^2 \times 1}{32} = 37.5 \text{公分}^2 \quad \#$$

$$4. \quad \text{幾何平均數} = \sqrt[4]{6.4 \times 0.9 \times 1.44 \times 4} = \sqrt[4]{(64 \times 0.1) \times (9 \times 0.1) \times (144 \times 0.01) \times 4}$$

$$= \sqrt[4]{2^6 \times 3^2 \times (3^2 \times 2^4) \times 2^2 \times 10^{-4}} = \sqrt[4]{2^{12} \times 3^4 \times 10^{-4}} = 2^3 \times 3 \times 10^{-1} = 2.4 \quad \#$$

$$5. \quad \text{最小值發生在 } x = \bar{x} = \frac{1+2+3+4}{4} = 2.5$$

$$\text{最小值} = f(\bar{x}) = (2.5-1)^2 + (2.5-2)^2 + (2.5-3)^2 + (2.5-4)^2 = 2.25 + 0.25 + 0.25 + 2.25 = 5 \quad \#$$

6.

當 $y_i = ax_i + b$ 時：

$$\text{則 } \bar{y} = a\bar{x} + b, \sigma_Y = |a|\sigma_X$$

因為 $y_i = -4x_i - 2$

$$\text{所以 } \bar{y} = -4\bar{x} - 2 = -4 \times 60 - 2 = -242$$

$$\text{而且 } \sigma_Y = |-4|\sigma_X = 4 \times 10 = 40$$

$$\text{所以 } (a, b) = (-242, 40) \#$$

$$7. \bar{x} = \frac{39+37+43+41+40}{5} = 40$$

$$\sigma = \sqrt{\frac{(-1)^2 + (-3)^2 + 3^2 + 1^2 + 0^2}{5}} = \sqrt{4} = 2$$

標準化數據：

$$39 \rightarrow \frac{39-40}{2} = -\frac{1}{2}$$

$$37 \rightarrow \frac{37-40}{2} = -\frac{3}{2}$$

$$43 \rightarrow \frac{43-40}{2} = \frac{3}{2}$$

$$41 \rightarrow \frac{41-40}{2} = \frac{1}{2}$$

$$40 \rightarrow \frac{40-40}{2} = 0$$

$$\text{Ans : } -\frac{1}{2}, -\frac{3}{2}, \frac{3}{2}, \frac{1}{2}, 0 \#$$

$$8. \bar{x} = \frac{1}{25} \sum_{i=1}^{25} x_i = \frac{1}{25} \times 100 = 4,$$

$$\text{標準差} = \sigma = \sqrt{\frac{1}{25} \times \left(\sum_{i=1}^{25} x_i^2 - 25\bar{x}^2 \right)} = \sqrt{\frac{1}{25} (1300 - 25 \times 4^2)} = \sqrt{\frac{900}{25}} = \sqrt{36} = 6 \#$$

$$9. 11 \text{ 個數字總和} = 75 \times 11 = 825$$

$$\text{剩下 } 10 \text{ 個數字總和} = 825 - 25 = 800$$

$$\text{剩下 } 10 \text{ 個數字平均} = \frac{800}{10} = 80 \#$$

10.

由平均數=7可以得知：

$$\frac{3+5+6+a+b}{5} = 7 \Rightarrow 14+a+b=35 \Rightarrow a+b=21 \Rightarrow a=21-b \dots\dots(1)$$

由標準差= $\sqrt{10}$ 可以得知：

$$\sqrt{\frac{3^2+5^2+6^2+a^2+b^2-5\times 7^2}{5}} = \sqrt{10} \Rightarrow 70+a^2+b^2-245=50 \Rightarrow a^2+b^2=50+245-70=225 \dots\dots(2)$$

將(1)代入(2)，得：

$$(21-b)^2+b^2=225 \Rightarrow 2b^2-42b+441=225 \Rightarrow 2b^2-42b+216=0 \Rightarrow b^2-21b+108=0$$

$$\Rightarrow (b-9)(b-12)=0 \Rightarrow b=9 \text{ or } b=12$$

①. $b=9$: $a=21-9=12$ ，不合(因為 $a \leq b$)

②. $b=12$: $a=21-12=9$ ，ok!!

所以 $(a,b)=(9,12)$ #