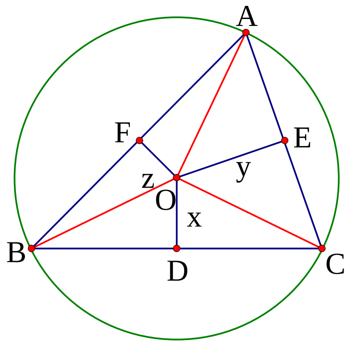
一、

1.(B)

2.(B)

解：如右圖所示，



$\angle BOC = 2\angle A$(∵對同弧的圓心角是圓周角的 2 倍)

$\Rightarrow \angle BOD = \angle A$(∵$\Delta BOC$ 是等腰三角形)

$\therefore x = \overline{OB}\cos\angle BOD = \overline{OB}\cos A$

同理，$y = \overline{OC}\cos B, z = \overline{OA}\cos C$

又 $\overline{OA} = \overline{OB} = \overline{OC} \Rightarrow x:y:z = \cos A:\cos B:\cos C$

故選(B)

二、

1.(A)(C)(D)(E)

2.(A)(D)(E)

解：(A)O: 由投影定理知，$c = a\cos B + b\cos A$

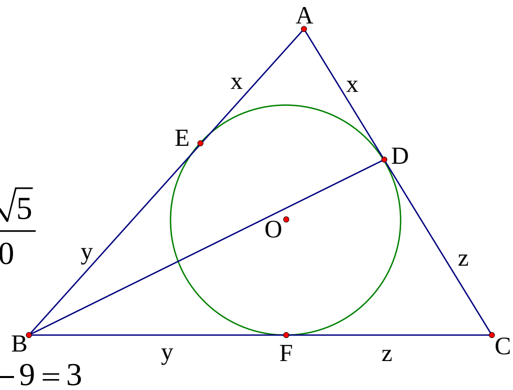
(B)×: 同理，$b = a\cos C + c\cos A$

(C)×: 同理，$a = b\cos C + c\cos B$

(D)O: 由餘弦定理知 $c^2 = a^2 + b^2 - 2ab\cos C \Rightarrow c = \sqrt{a^2 + b^2 - 2ab\cos C}$

(E)O: 由正弦定理知 $\dfrac{a}{\sin A} = \dfrac{c}{\sin C} \Rightarrow c = a\dfrac{\sin C}{\sin A}$

3.(A)(B)(C)



解：(A)O: $s = \dfrac{7+8+9}{2} = 12$, 由海龍公式

$\Delta ABC = \sqrt{12\cdot 5\cdot 4\cdot 3} = 12\sqrt{5}$

(B)O: $\Delta ABC = \dfrac{abc}{4R} \Rightarrow R = \dfrac{9\cdot 7\cdot 8}{4\cdot 12\sqrt{5}} = \dfrac{21}{2\sqrt{5}} = \dfrac{21\sqrt{5}}{10}$

(C)O: $\Delta ABC = rs \Rightarrow 12\sqrt{5} = 12r \therefore r = \sqrt{5}$

(D)×: 由右圖及計算結果可得 $x = s - \overline{BC} = 12 - 9 = 3$

又 $\cos A = \dfrac{64 + 9 - \overline{BD}^2}{2\cdot 8\cdot 3} = \dfrac{64 + 56 - 81}{2\cdot 8\cdot 7}$

$\Rightarrow \dfrac{73 - \overline{BD}^2}{3} = \dfrac{39}{7} \Rightarrow 7\overline{BD}^2 = 511 - 117 = 394$

$\therefore \overline{BD} = \sqrt{\dfrac{394}{7}}$

(E)×: 我們也可以得到 $z = 4$

$\overline{OA}=\sqrt{9+5}=\sqrt{14}, \overline{OC}=\sqrt{16+5}=\sqrt{21}$

$\therefore \overline{OA}:\overline{OC}=\sqrt{14}:\sqrt{21}=\sqrt{2}:\sqrt{3}$

故選(A)(B)(C)

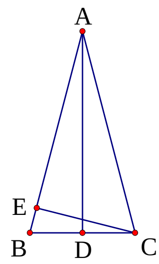
三、

1.(1)7 (2)$\frac{1}{2}$ (3)$\frac{39}{4}\sqrt{3}$ (4)$\frac{49\pi}{3}$

2.(1)$4\sqrt{6}$ (2)$\frac{\sqrt{6}}{2}$

3.$-\frac{\sqrt{6}}{12}$

4.$\frac{1}{4}$



解：∵底邊的比會是高的倒數比

$\Rightarrow \frac{\overline{AB}}{\overline{BC}}=2$, 令 $\overline{BC}=2 \Rightarrow \overline{AB}=4, \overline{BD}=1$

$\therefore \cos B=\frac{\overline{BD}}{\overline{AB}}=\frac{1}{4}$

5.$\frac{7}{10}$

解：$\Delta ABD=\frac{1}{2}\cdot 6\cdot \overline{AD}\sin\angle BAD$

$\Delta ACD=\frac{1}{2}\cdot 7\cdot \overline{AD}\sin\angle DAC$

又 $\frac{\Delta ABD}{\Delta ACD}=\frac{3}{5}\Rightarrow \frac{3}{5}=\frac{6\sin\angle BAD}{7\sin\angle DAC}$

$\therefore \frac{\sin\angle BAD}{\sin\angle DAC}=\frac{7}{10}$

6.

解：如圖，在 ΔAMB 及 ΔMNB 中分別應用餘弦定理，得

$$\overline{MB}^2=16+80-32\sqrt{5}\cos A=96-32\sqrt{5}\cos A$$

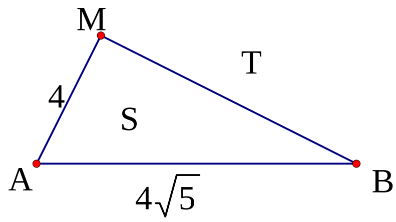
及

$$\overline{MB}^2=16+16-32\cos N=32-32\cos N$$

兩式相減，得 $\cos N=\sqrt{5}\cos A-2$. 由三角形的面積公式。有

$S^2+T^2=(8\sqrt{5}\sin A)^2+(8\sin N)^2$

$$=320(1-\cos^2 A)+64(1-\cos^2 N)$$
$$=384-320\cos^2 A-64(\sqrt{5}\cos A-2)^2$$
$$=-640\left(\cos A-\frac{1}{\sqrt{5}}\right)^2+256.$$

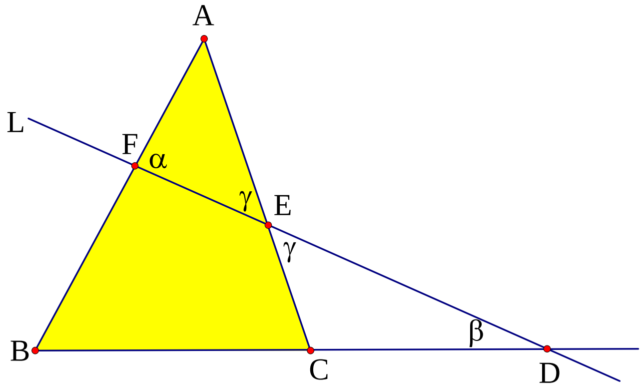


(1)

四、

2.

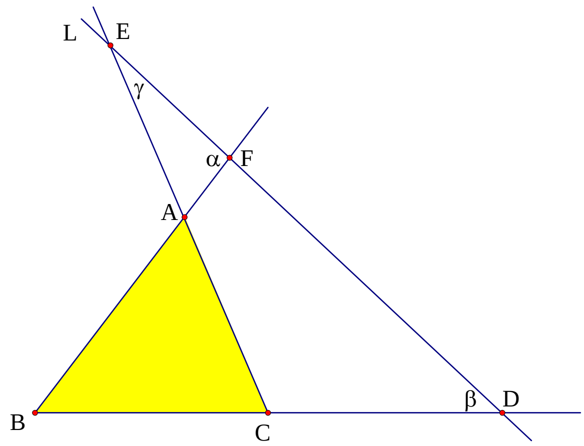
證明：設截線 L 與三條直線 $\overleftrightarrow{AB},\overleftrightarrow{BC},\overleftrightarrow{CA}$ 的夾角為 α,β,γ, 如下圖所示。



(1) 由正弦定理得

$$\frac{\overline{AF}}{\overline{EA}}=\frac{\sin\gamma}{\sin\alpha}$$
$$\frac{\overline{BD}}{\overline{FB}}=\frac{\sin(\pi-\alpha)}{\sin\beta}=\frac{\sin\alpha}{\sin\beta}$$
$$\frac{\overline{CE}}{\overline{DC}}=\frac{\sin\beta}{\sin\gamma}$$
$$\therefore\frac{\overline{AF}}{\overline{FB}}\cdot\frac{\overline{BD}}{\overline{DC}}\cdot\frac{\overline{CE}}{\overline{EA}}=\frac{\overline{AF}}{\overline{EA}}\cdot\frac{\overline{BD}}{\overline{FB}}\cdot\frac{\overline{CE}}{\overline{DC}}$$
$$=\frac{\sin\gamma}{\sin\alpha}\cdot\frac{\sin\alpha}{\sin\beta}\cdot\frac{\sin\beta}{\sin\gamma}=1$$

即 $\frac{\overline{AF}}{\overline{FB}}\cdot\frac{\overline{BD}}{\overline{DC}}\cdot\frac{\overline{CE}}{\overline{EA}}=1.$

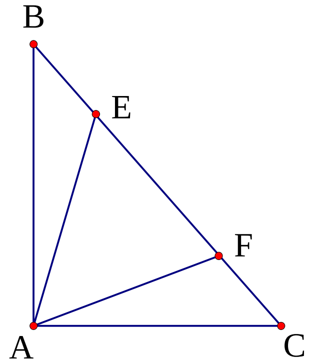


(2) 由正弦定理得

$$\frac{\overline{AF}}{\overline{AE}}=\frac{\sin\gamma}{\sin\alpha}$$
$$\frac{\overline{BD}}{\overline{FB}}=\frac{\sin(\pi-\alpha)}{\sin\beta}=\frac{\sin\alpha}{\sin\beta}$$
$$\frac{\overline{CE}}{\overline{DC}}=\frac{\sin\beta}{\sin\gamma}$$
$$\therefore\frac{\overline{AF}}{\overline{FB}}\cdot\frac{\overline{BD}}{\overline{DC}}\cdot\frac{\overline{CE}}{\overline{EA}}=\frac{\overline{AF}}{\overline{AE}}\cdot\frac{\overline{BD}}{\overline{FB}}\cdot\frac{\overline{CE}}{\overline{DC}}$$
$$=\frac{\sin\gamma}{\sin\alpha}\cdot\frac{\sin\alpha}{\sin\beta}\cdot\frac{\sin\beta}{\sin\gamma}=1$$

即 $\frac{\overline{AF}}{\overline{FB}}\cdot\frac{\overline{BD}}{\overline{DC}}\cdot\frac{\overline{CE}}{\overline{EA}}=1.$

3.



證明：在 ΔABE 及 ΔACF 中分別應用餘弦定理，得

$$\overline{AE}^2 = \overline{AB}^2 + \overline{BE}^2 - 2\overline{AB}\,\overline{BE}\cos B \qquad (1)$$

及

$$\overline{AF}^2 = \overline{AC}^2 + \overline{CF}^2 - 2\overline{AC}\,\overline{CF}\cos C \qquad (2)$$

由題設 $\overline{BE} = \overline{CF} = \frac{1}{4}\overline{BC}$, 於是 $(1)+(2)$ 得

$$\begin{aligned}
\overline{AE}^2 + \overline{AF}^2 &= (\overline{AB}^2 + \overline{AC}^2) + \frac{1}{8}\overline{BC}^2 - \frac{1}{2}\overline{BC}(\overline{AB}\cos B + \overline{AC}\cos C) \\
&= \overline{BC}^2 + \frac{1}{8}\overline{BC}^2 - \frac{1}{2}\overline{BC}\left(\overline{AB}\,\frac{\overline{AB}}{\overline{BC}} + \overline{AC}\,\frac{\overline{AC}}{\overline{BC}}\right) \\
&= \overline{BC}^2 + \frac{1}{8}\overline{BC}^2 - \frac{1}{2}\overline{BC}^2 \\
&= \frac{9}{8}\overline{BC}^2 - \frac{4}{8}\overline{BC}^2 = \frac{5}{8}\overline{BC}^2.
\end{aligned}$$