

例題演練

例題 10. $25(\sqrt{3} + 1)$

課後練習

9.4.5

$$10. 100\sqrt{19}$$

$$\text{解: } \cos 120^\circ = \frac{200^2 + 300^2 - \overline{AB}^2}{2 \cdot 200 \cdot 300}$$

$$\begin{aligned}\overline{AB}^2 &= 200^2 + 300^2 + 200 \cdot 300 = 190000 \\ \therefore \overline{AB} &= 100\sqrt{19}\end{aligned}$$



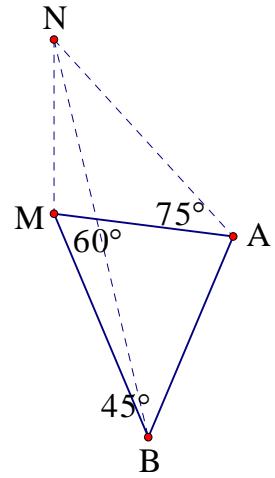
$$11. 600$$

$$12. 150\sqrt{6 - 3\sqrt{3}}$$

解: 1. 由圖知: $\overline{BM} = 150$

$$\begin{aligned}2. \tan 75^\circ &= \frac{150}{\overline{AM}} \Rightarrow \overline{AM} = 150 \cot 75^\circ \\ &= 150 \tan 75^\circ = 150(2 - \sqrt{3})\end{aligned}$$

$$\begin{aligned}3. \cos 60^\circ &= \frac{[150(2 - \sqrt{3})]^2 + 150^2 - \overline{AB}^2}{2 \cdot 150(2 - \sqrt{3}) \cdot 150} \\ \overline{AB} &= 150\sqrt{6 - 3\sqrt{3}}\end{aligned}$$



$$13. \frac{3(3\sqrt{3} + 2)}{23}$$

$$14. (1) 3\sqrt{7} (\text{浬/時}) (2) \frac{\sqrt{21}}{7} \text{浬}$$

解: (1) 由 $\overline{AB}^2 = \overline{OA}^2 + \overline{OB}^2 - 2\overline{OA}\overline{OB} \cos 120^\circ$

$$= 2^2 + 1^2 - 2 \cdot 2 \cdot 1 \left(-\frac{1}{2} \right) = 7$$

$$\therefore \overline{AB} = \sqrt{7}$$

$$\therefore \text{漁船之時速為 } \frac{\sqrt{7}}{\frac{1}{3}} = 3\sqrt{7} (\text{浬/時})$$

(2) 設漁船離觀測點 O 之最近距離為 x

$$\because \Delta AOB \text{ 中} : \frac{1}{2} \cdot 2 \cdot 1 \sin 120^\circ = \frac{1}{2} \overline{AB}x \Rightarrow \sqrt{3} = \sqrt{7}x \Rightarrow x = \frac{\sqrt{3}}{\sqrt{7}} = \frac{\sqrt{21}}{7}$$

$$15. \frac{15\sqrt{2}}{2}, -1 + \frac{2\sqrt{3}}{3}$$

解：(1)由題意作圖如右，令塔高 $\overline{CH} = h$

$$\Delta ACH \text{ 中} : \tan 75^\circ = \frac{h}{AC}$$

$$\Rightarrow \overline{AC} = (2 - \sqrt{3})h$$

$\uparrow \tan 75^\circ = 2 + \sqrt{3}$

$$\Delta BCH \text{ 中} : \tan 30^\circ = \frac{h}{BC} \Rightarrow \overline{BC} = \sqrt{3}h, \text{ 又 } \overline{AB} = \frac{30}{\sqrt{3} + 1}$$

(2) 在 ΔABC (直角 Δ)，由商高定理

$$\overline{BC}^2 = \overline{AC}^2 + \overline{AB}^2 \Rightarrow 3h^2 = (7 - 4\sqrt{3})h^2 + \frac{900}{\sqrt{3} + 1} \Rightarrow \text{塔高 } h = \frac{15\sqrt{2}}{2}$$

$$(3) \text{直角 } \Delta ABC \text{ 中} : \cos(\angle ACB) = \frac{\overline{AC}}{\overline{BC}} = \frac{(2 - \sqrt{3})h}{\sqrt{3}h} = \frac{2\sqrt{3} - 3}{3} = -1 + \frac{2\sqrt{3}}{3}$$