一、

1. 答案：(B)

解：$a=\cos 340^\circ=\cos(360^\circ-20^\circ)=\cos 20^\circ>\cos 23^\circ>\sin 23^\circ$

$b=\sin 157^\circ=\sin(180^\circ-23^\circ)=\sin 23^\circ$

$c=\tan 225^\circ=\tan(180^\circ+45^\circ)=\tan 45^\circ=1$

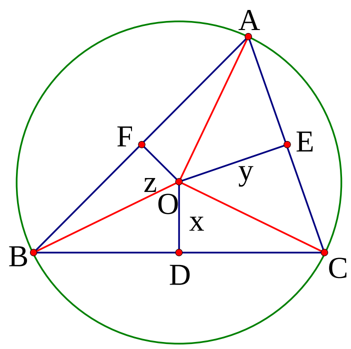
$d=\cot 220^\circ=\cot(180^\circ+40^\circ)=\cot 40^\circ>\cot 45^\circ=1$

$e=\csc 503^\circ=\csc 143^\circ=\csc(180^\circ-43^\circ)=\csc 43^\circ>\cot 40^\circ$

$\therefore b<a<c<d<e$　故選(B)

2. 答案：(B)

解：如右圖所示，



$\angle BOC=2\angle A$($\because$ 對同弧的圓心角是圓周角的 2 倍)

$\Rightarrow \angle BOD=\angle A$($\because \Delta BOC$ 是等腰三角形)

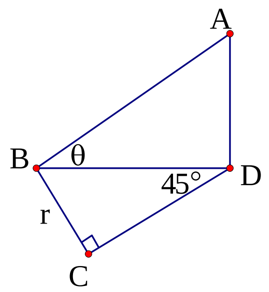
$\therefore x=\overline{OB}\cos\angle BOD=\overline{OB}\cos A$

同理，$y=\overline{OC}\cos B, z=\overline{OA}\cos C$

又 $\overline{OA}=\overline{OB}=\overline{OC}\Rightarrow x:y:z=\cos A:\cos B:\cos C$

故選(B)

3. 答案：(C)



解：$\overline{BD}=r\sec 45^\circ=\sqrt{2}r$

$\overline{AD}=\overline{BD}\tan\theta=\sqrt{2}r\tan\theta$

故選(C)

二、

1. 答案：(A)(D)(E)

解：(A)O:由投影定理知，$c=a\cos B+b\cos A$

(B)×:同理，$b=a\cos C+c\cos A$

(C)×:同理，$a=b\cos C+c\cos B$

(D)O:由餘弦定理知 $c^2=a^2+b^2-2ab\cos C\Rightarrow c=\sqrt{a^2+b^2-2ab\cos C}$

(E)O:由正弦定理知 $\dfrac{a}{\sin A}=\dfrac{c}{\sin C}\Rightarrow c=a\dfrac{\sin C}{\sin A}$

2. 答案：(A)(B)(C)(E)

解：(A)O:$\because \angle ACD=\angle CAB\Rightarrow \overline{AB}\,//\,\overline{CD}$(內錯角相等)

$$\Rightarrow \frac{\overline{AO}}{\overline{CO}}=\frac{\overline{BO}}{\overline{DO}}\therefore \overline{AO}\times\overline{DO}=\overline{BO}\times\overline{CO}$$

(B)O: $\because \overline{AD}=\overline{AC}=\overline{AB}=\sqrt{2} \Rightarrow A$ 為 ΔBCD 的外接圓圓心

(C)O: $\because \angle ACD=\angle CAB$

$\Rightarrow \cos\angle ACD=\cos\angle CAB=\dfrac{2+2-3}{2\cdot\sqrt{2}\cdot\sqrt{2}}=\dfrac{1}{4}$

(D)×: 由(2)知 ΔBCD 的外接圓半徑 $=\sqrt{2}$

由正弦定理，$\dfrac{\overline{BC}}{\sin\angle BDC}=2\sqrt{2}$

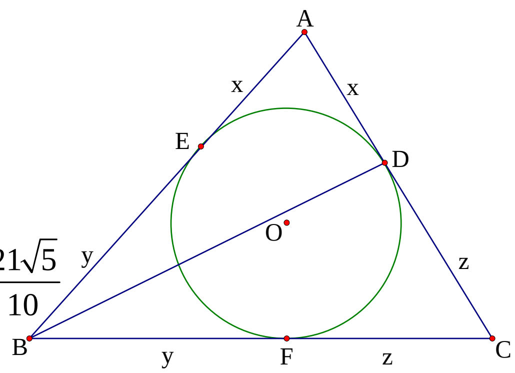
$\therefore \sin\angle BDC=\dfrac{\overline{BC}}{2\sqrt{2}}=\dfrac{\sqrt{3}}{2\sqrt{2}}=\dfrac{\sqrt{6}}{4}$

(E)O: $\angle ABC+\angle BCD=180^{\circ} \Rightarrow \sin\angle ABC=\sin\angle BCD$

由正弦定理知 $\dfrac{\overline{BD}}{\sin\angle BCD}=2\sqrt{2} \Rightarrow \overline{BD}=2\sqrt{2}\sin\angle BCD=2\sqrt{2}\sin\angle ABC$

故選(A)(B)(C)(E)

3. 答案：(A)(B)(C)



解：(A)O: $s=\dfrac{7+8+9}{2}=12$, 由海龍公式

$\Delta ABC=\sqrt{12\cdot5\cdot4\cdot3}=12\sqrt{5}$

(B)O: $\Delta ABC=\dfrac{abc}{4R} \Rightarrow R=\dfrac{9\cdot7\cdot8}{4\cdot12\sqrt{5}}=\dfrac{21}{2\sqrt{5}}=\dfrac{21\sqrt{5}}{10}$

(C)O: $\Delta ABC=rs \Rightarrow 12\sqrt{5}=12r \therefore r=\sqrt{5}$

(D)×: 由右圖及計算結果可得 $x=s-\overline{BC}=12-9=3$

又 $\cos A=\dfrac{64+9-\overline{BD}^2}{2\cdot8\cdot3}=\dfrac{64+56-81}{2\cdot8\cdot7}$

$\Rightarrow \dfrac{73-\overline{BD}^2}{3}=\dfrac{39}{7} \Rightarrow 7\overline{BD}^2=511-117=394$

$\therefore \overline{BD}=\sqrt{\dfrac{394}{7}}$

(E)×: 我們也可以得到 $z=4$

$\overline{OA}=\sqrt{9+5}=\sqrt{14}, \overline{OC}=\sqrt{16+5}=\sqrt{21}$

$\therefore \overline{OA}:\overline{OC}=\sqrt{14}:\sqrt{21}=\sqrt{2}:\sqrt{3}$

故選(A)(B)(C)

三、

1. 答案：2

解：$1=(\sin^2\theta+\cos^2\theta)^3=\sin^6\theta+\cos^6\theta+3\sin^2\theta\cos^2\theta(\sin^2\theta+\cos^2\theta)$

$=\sin^6\theta+\cos^6\theta+3\sin^2\theta\cos^2\theta$

$\Rightarrow \sin^6\theta+\cos^6\theta=1-3\sin^2\theta\cos^2\theta$

$1=(\sin^2\theta+\cos^2\theta)^2=\sin^4\theta+\cos^4\theta+2\sin^2\theta\cos^2\theta$

$\Rightarrow \sin^4\theta+\cos^4\theta=1-2\sin^2\theta\cos^2\theta$

原式$=1-3\sin^2\theta\cos^2\theta+1-2\sin^2\theta\cos^2\theta+5\sin^2\theta\cos^2\theta=2$

2. 答案：$\frac{1}{4}$

解：∵底邊的比會是高的倒數比

$\Rightarrow \frac{\overline{AB}}{\overline{BC}}=2$, 令 $\overline{BC}=2\Rightarrow \overline{AB}=4, \overline{BD}=1$

$\therefore \cos B=\frac{\overline{BD}}{\overline{AB}}=\frac{1}{4}$

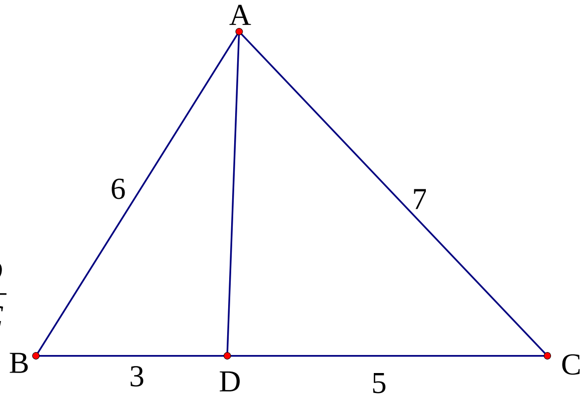
3. 答案：$\sqrt{65}$

解：令 $\overline{AB}=5x\Rightarrow \overline{BH}=3x, \overline{AH}=4x, \overline{CH}=2x$

$\Rightarrow \overline{BM}=\overline{CM}\Rightarrow 3x-1=2x+1\Rightarrow x=2$

$\overline{AH}=4x=8 \therefore \overline{AM}=\sqrt{1+64}=\sqrt{65}$

4. 答案：$\frac{7}{10}$



解：$\Delta ABD=\frac{1}{2}\cdot 6\cdot \overline{AD}\sin\angle BAD$

$\Delta ACD=\frac{1}{2}\cdot 7\cdot \overline{AD}\sin\angle DAC$

又 $\frac{\Delta ABD}{\Delta ACD}=\frac{3}{5}\Rightarrow \frac{3}{5}=\frac{6\sin\angle BAD}{7\sin\angle DAC}$

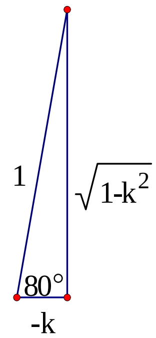
$\therefore \frac{\sin\angle BAD}{\sin\angle DAC}=\frac{7}{10}$

5. 答案：$\frac{9}{7}$

解：若 $\cos\theta=-\frac{4}{5}\Rightarrow \sin\theta=\pm\sqrt{1-\cos^2\theta}=\pm\frac{3}{5}=\frac{y}{r}=\frac{-4}{r}$(正不合)

$\therefore r=\frac{20}{3}\Rightarrow x=r\cos\theta=\frac{20}{3}(-\frac{4}{5})=-\frac{16}{3}$

$\therefore$所求$=\frac{3(-\frac{16}{3})+4}{-\frac{16}{3}-4}=\frac{-48+12}{-16-12}=\frac{-36}{-28}=\frac{9}{7}$



6. 答案：$-\sqrt{1-k^2}$

解：$k=\cos 100^\circ=\cos(180^\circ-80^\circ)=-\cos 80^\circ$

依此條件下我們可以畫出直角三角形

$\sin 280^\circ=\sin(360^\circ-80^\circ)=-\sin 80^\circ=-\sqrt{1-k^2}$

7.答案：$\dfrac{\sqrt{5}}{10}$

解：設另一根為α

由根與係數知，$(2+\sqrt{5})\alpha=1\Rightarrow\alpha=\dfrac{1}{2+\sqrt{5}}=\sqrt{5}-2$

再由根與係數知$\tan\theta+\cot\theta=\sqrt{5}-2+\sqrt{5}+2=2\sqrt{5}$

又$\sin\theta\cos\theta=\dfrac{1}{\tan\theta+\cot\theta}=\dfrac{1}{2\sqrt{5}}=\dfrac{\sqrt{5}}{10}$

8. 答案：120°

解：由根與係數知：

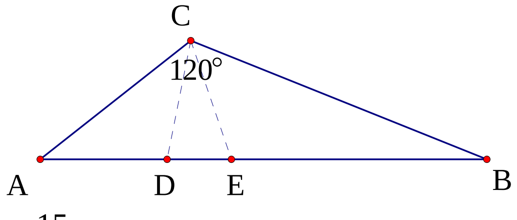
$$\begin{cases} a+b=2\sqrt{3} \\ ab=2 \end{cases}$$

$$\cos C=\frac{a^2+b^2-c^2}{2ab}=\frac{(a+b)^2-2ab-10}{2\cdot 2}=\frac{12-4-10}{4}=-\frac{1}{2}$$

$\therefore \angle C=120^\circ$

9. 答案：$\dfrac{15}{28}\sqrt{3}$

解：由餘弦定理：



$\overline{AB}^2=3^2+5^2-2\cdot 3\cdot 5\cdot\cos 120^\circ\Rightarrow\overline{AB}=7$

$\overline{DE}=\overline{AE}+\overline{BD}-\overline{AB}$

$=3+5-7=1$

$\therefore \Delta CDE=\dfrac{1}{7}\Delta ABC=\dfrac{1}{7}\left(\dfrac{1}{2}\cdot 3\cdot 5\cdot\sin 120^\circ\right)=\dfrac{15}{28}\sqrt{3}$

10. 答案：-0.9766

解：$\because \sin 12^\circ 20'=0.2136$

$\sin\theta=0.2150$

$\therefore$ 令$\sin\alpha=0.2150$利用內插法

$$\frac{\alpha-12^{\circ}20'}{12^{\circ}30'-12^{\circ}20'}=\frac{0.2150-0.2136}{0.2164-0.2136}=\frac{1}{2}$$

$\therefore \alpha=12^{\circ}25', \because 90^{\circ}<\theta<180^{\circ}$,

$\therefore \theta=180^{\circ}-\alpha=167^{\circ}35'$

$\cos\theta=\cos 167^{\circ}35'=\cos(180-12^{\circ}25')=-\cos 12^{\circ}25'$

$\cos 12^{\circ}25'=x$

$$\frac{12^{\circ}25'-12^{\circ}20'}{12^{\circ}30'-12^{\circ}20'}=\frac{x-0.9769}{0.9763-0.9769}$$

$\therefore x=0.9766$

$\Rightarrow \cos\theta=-x=-0.9766$

11. 答案：$k=2$ 或 $k\geq 4$

解：因為是兩鄰邊與一對角，因此有兩種可能

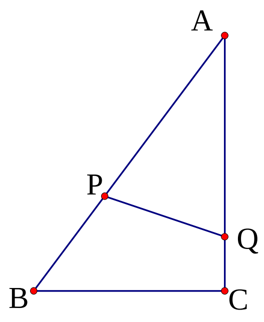
一種是 $\angle B$ 的對邊 $\overline{AC}$ 對到較大的邊

即 $\overline{AC}=k>4=\overline{AB}$

或是讓 $\overline{AB}$ 成為直角三角形的斜邊

那麼 $\overline{AC}=\overline{AB}\sin 30^{\circ}=4\cdot\frac{1}{2}=2 \therefore k=2$ 或 $k\geq 4$

12. 答案：2



解：由題意：$\Delta ABC=\frac{1}{2}\overline{AP}\cdot\overline{AQ}\sin A=\frac{1}{2}\cdot\frac{1}{2}\cdot 3\cdot 4$

$\Rightarrow \overline{AP}\cdot\overline{AQ}\cdot\frac{3}{5}=\frac{1}{2}\cdot 3\cdot 4 \therefore \overline{AP}\cdot\overline{AQ}=10$

由餘弦定理知：

$$\overline{PQ}^2=\overline{AP}^2+\overline{AQ}^2-2\overline{AP}\cdot\overline{AQ}\cdot\cos A\geq 2\overline{AP}\cdot\overline{AQ}-2\cdot 10\cdot\frac{4}{5}=20-16=4$$

(算幾定理)

$\therefore \overline{PQ}\geq 2$

13. 答案：720

解：$\sum_{i=1}^{360}\overline{P_{180}P_i}^2=\overline{P_{180}P_1}^2+\overline{P_{180}P_2}^2+\cdots+\overline{P_{180}P_{179}}^2+\overline{P_{180}P_{180}}^2+\overline{P_{180}P_{181}}^2+\cdots+\overline{P_{180}P_{360}}^2$

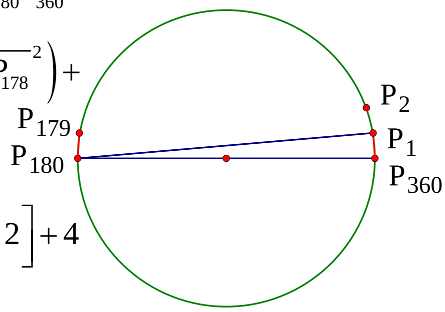
$$=2\left[\overline{P_{180}P_1}^2+\overline{P_{180}P_2}^2+\cdots+\overline{P_{180}P_{179}}^2\right]+\overline{P_{180}P_{360}}^2$$

$$=2\left[\left(\overline{P_{180}P_1}^2+\overline{P_{180}P_{179}}^2\right)+\left(\overline{P_{180}P_2}^2+\overline{P_{180}P_{178}}^2\right)+\right.$$

$$\left.\cdots+\left(\overline{P_{180}P_{89}}^2+\overline{P_{180}P_{91}}^2\right)+\overline{P_{180}P_{90}}^2\right]+4$$

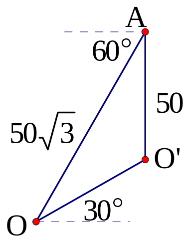
$$=2\left[\overline{P_{180}P_{360}}^2+\overline{P_{180}P_{360}}^2+\cdots+\overline{P_{180}P_{360}}^2+2\right]+4$$

$$=2(89\times4+2)+4=720$$



14. 答案：5

解：如右圖，由餘弦定理知：



$$\cos O=\cos30^\circ=\frac{\overline{OO'}^2+7500-2500}{2\overline{OO'}\cdot50\sqrt{3}}$$

$$\Rightarrow \overline{OO'}^2-150\overline{OO'}+5000=0$$

$$(\overline{OO'}-50)(\overline{OO'}-100)=0$$

$\therefore \overline{OO'}=50$(100 不合，取較小的數)

又$\dfrac{50}{10}=5$ ∴下午 5 時會進入暴風圈