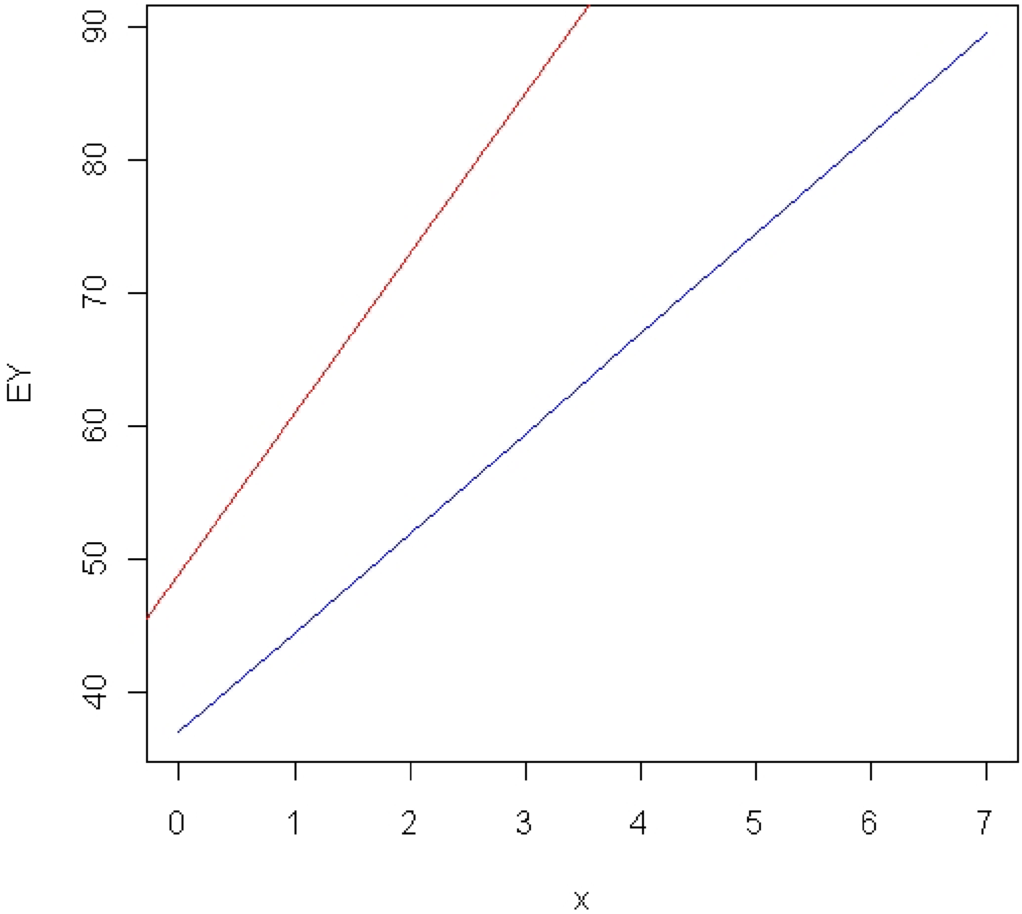
林柏佐

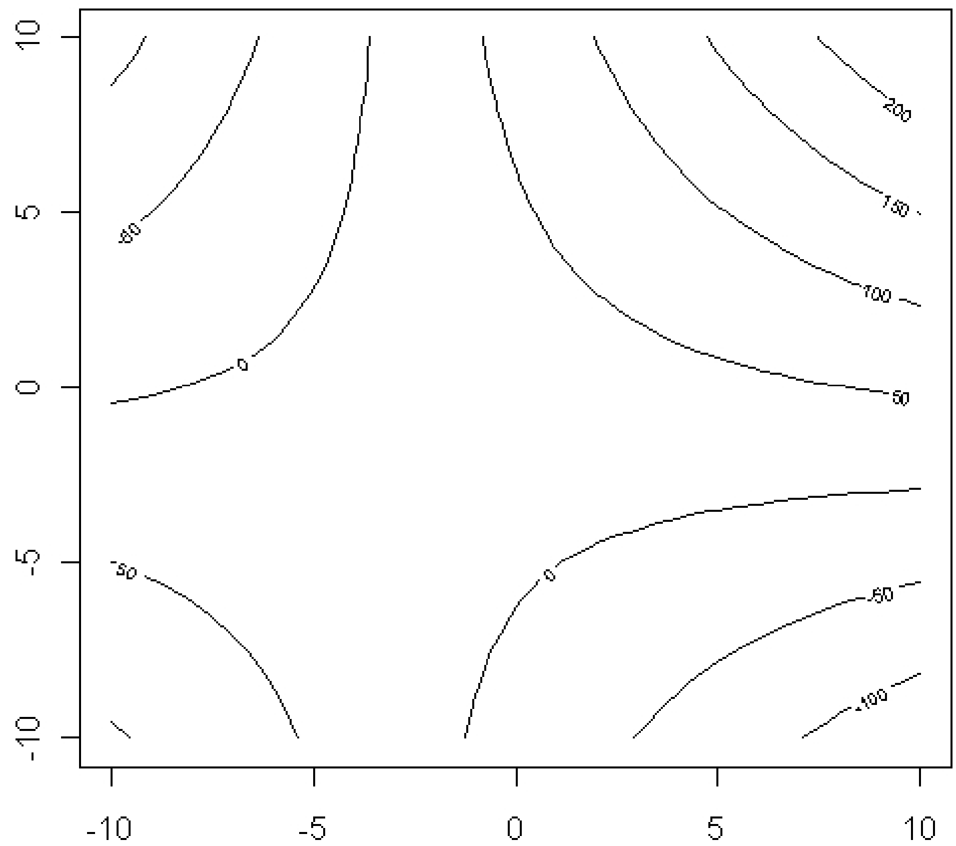
1.

a.



從圖形來看，我們知道 X_1 與 X_2 interaction 的影響在於 slope 的變化，當其中一個增加 1 unit, 另一個的 slope 相對應增加 1.5 unit.

b.從圖形來看，X_1 與 X_2 interaction 的影響在於讓 $E[Y]$ 的變化，會隨著 X_1 與 X_2 的變化而增加其變化量，而把它從 linear 的變化，轉變為 quadratic 的變化



2.

(a) $X^T X = \begin{bmatrix} 20 & 10 \\ 10 & 10 \end{bmatrix}$

(b) $(X^T X)^{-1} = \dfrac{1}{100}\begin{bmatrix} 10 & -10 \\ -10 & 20 \end{bmatrix} = \dfrac{1}{10}\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

$$\Rightarrow \mathbf{b} = (X^T X)^{-1} X^T Y = \frac{1}{10}\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} Y_1 + Y_2 + \cdots + Y_{20} \\ Y_1 + Y_2 + \cdots + Y_{10} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{Y_{11} + Y_{12} + \cdots + Y_{20}}{10} \\ \dfrac{Y_1 + Y_2 + \cdots + Y_{10}}{10} - \dfrac{Y_{11} + Y_{12} + \cdots + Y_{20}}{10} \end{bmatrix}$$

$$= \begin{bmatrix} 22.1 \\ 16.7 - 22.1 \end{bmatrix} = \begin{bmatrix} 22.1 \\ -5.4 \end{bmatrix}$$

(c) $H = X(X^T X)^{-1} X^T$

$$
=\begin{bmatrix}1 & 1\\ 1 & 1\\ \vdots & \vdots\\ 1 & 1\\ 1 & 0\\ 1 & 0\\ \vdots & \vdots\\ 1 & 0\end{bmatrix}\cdot\frac{1}{10}\begin{bmatrix}1 & -1\\ -1 & 2\end{bmatrix}\begin{bmatrix}1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1\\ 1 & 1 & \cdots & 1 & 0 & 0 & \cdots & 0\end{bmatrix}
$$

$$
=\frac{1}{10}\begin{bmatrix}0 & 1\\ 0 & 1\\ \vdots & \vdots\\ 0 & 1\\ 1 & -1\\ 1 & -1\\ \vdots & \vdots\\ 1 & -1\end{bmatrix}\begin{bmatrix}1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1\\ 1 & 1 & \cdots & 1 & 0 & 0 & \cdots & 0\end{bmatrix}
$$

$$
=\frac{1}{10}\begin{bmatrix}1 & 1 & \cdots & 1 & 0 & 0 & \cdots & 0\\ 1 & 1 & \cdots & 1 & 0 & 0 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 1 & 1 & \cdots & 1 & 0 & 0 & \cdots & 0\\ 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & 1\\ 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & 1\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & 1\end{bmatrix}
$$

$$\hat{Y} = HY = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ \vdots & \vdots \\ 1 & 1 \\ 1 & 0 \\ 1 & 0 \\ \vdots & \vdots \\ 1 & 0 \end{bmatrix} \cdot \frac{1}{10}\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 0 & 1 \\ 0 & 1 \\ \vdots & \vdots \\ 0 & 1 \\ 1 & -1 \\ 1 & -1 \\ \vdots & \vdots \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 1 & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 \\ 1 & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & \cdots & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & 1 \\ 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 & 1 & \cdots & 1 \end{bmatrix}\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_{10} \\ Y_{11} \\ Y_{12} \\ \vdots \\ Y_{20} \end{bmatrix} = \begin{bmatrix} 16.7 \\ 16.7 \\ \vdots \\ 16.7 \\ 22.1 \\ 22.1 \\ \vdots \\ 22.1 \end{bmatrix}$$

$$\text{SSE} = \sum_{i=1}^{20}(Y_i - \hat{Y})^2 = \sum_{i=1}^{10}(Y_i - 16.7)^2 + \sum_{i=11}^{20}(Y_i - 22.1)^2 = 1535$$

$$\text{SSR} = \sum_{i=1}^{20}(\hat{Y} - \overline{Y})^2 = \sum_{i=1}^{10}(16.7 - 19.4)^2 + \sum_{i=11}^{20}(22.1 - 19.4)^2 = 20 \cdot 2.7^2 = 145.8$$

3.

(a) $[g'(\eta)\cdot\eta^k]^2 = c \Rightarrow g'(\eta) = \sqrt{c}\eta^{-k} \Rightarrow g(\eta) = \sqrt{c}(1-k)\eta^{1-k} + d = \dfrac{\sqrt{c}(1-k)}{\eta^{k-1}} + d,$

取 $c = \dfrac{1}{(1-k)^2}, d = 0 \therefore$ transformation 為 $\dfrac{1}{Y^{k-1}}$.

(b) $[g'(\eta)\cdot e^{\eta}]^2 = c \Rightarrow g'(\eta) = \sqrt{c}e^{-\eta} \Rightarrow g(\eta) = -\sqrt{c}e^{-\eta} + d = \dfrac{-\sqrt{c}}{e^{\eta}} + d,$

取 $c = 1, d = 0 \therefore$ transformation 為 $\dfrac{-1}{e^Y}$.

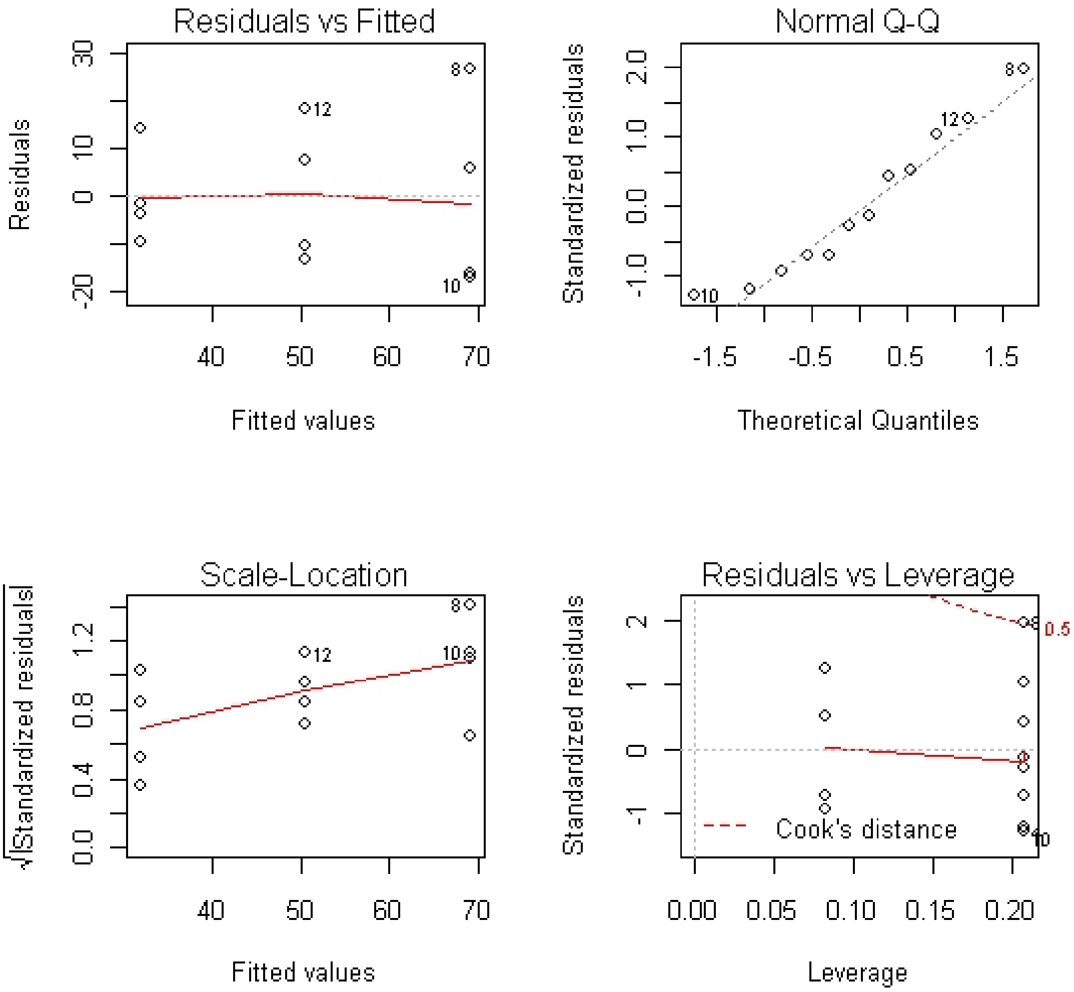
(c) $\left[\dfrac{g'(\eta)}{\log(\eta)}\right]^2 = c \Rightarrow g'(\eta) = \sqrt{c}\log(\eta) \Rightarrow g(\eta) = \sqrt{c}(\eta\log(\eta) - \eta) + d$

取 $c = 1, d = 0 \therefore$ transformation 為 $Y\log(Y) - Y$.

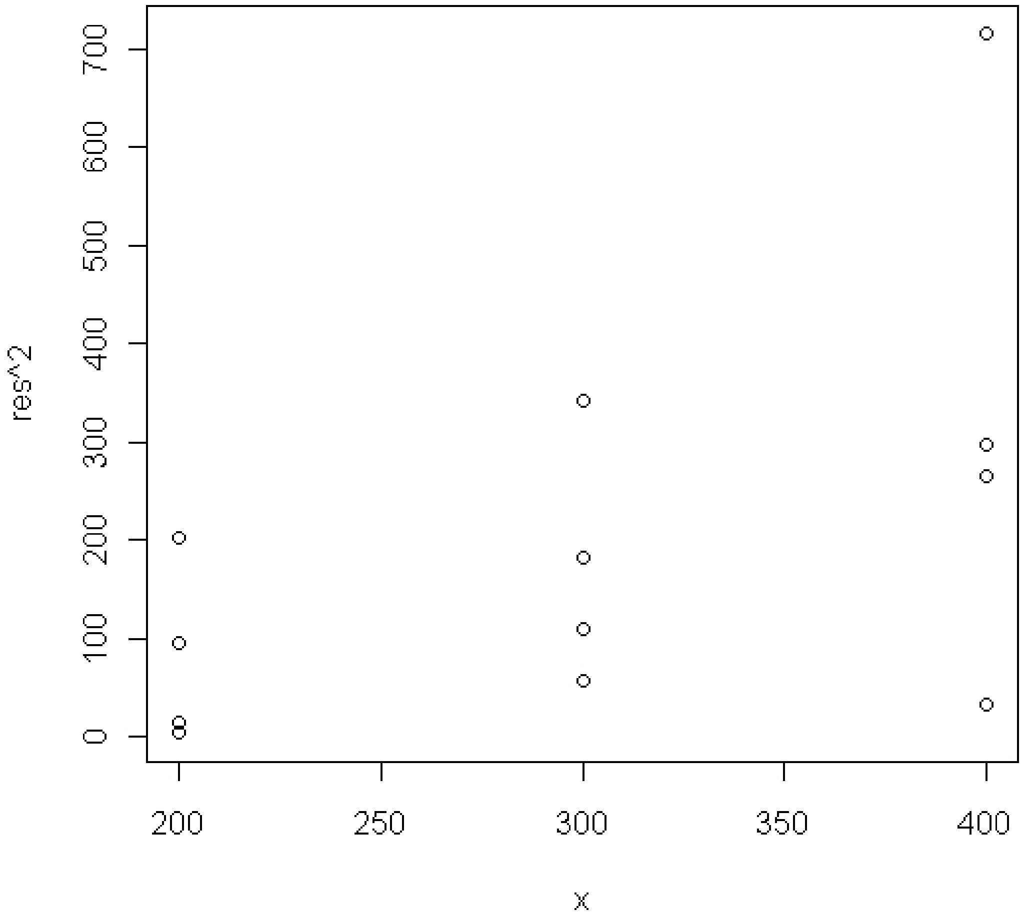
5.

a.其 regression function 為 $\hat{Y} = -5.75 + 0.1875x$

從此圖形來看，我們可知其 error 不是 equal variance.



c.從圖形來看，我們可以猜測 error terms 其 variance 與 x^2 成正比。



d.

	x	Y	res	res^2	weights
1	200	28	-3.8	14.1	0.0146
2	400	75	5.8	33.1	0.0032
3	300	37	-13.5	182.2	0.0052
4	400	53	-16.2	264.1	0.0032
5	200	22	-9.8	95.1	0.0146
6	300	58	7.5	56.2	0.0052
7	300	40	-10.5	110.2	0.0052
8	400	96	26.8	715.6	0.0032
9	200	46	14.2	203.1	0.0146
10	400	52	-17.2	297.6	0.0032
11	200	30	-1.7	3.1	0.0146
12	300	69	18.5	342.2	0.0052

e.weighted least squares $\hat{Y} = -6.23322 + 0.18911x$ 與(a)略有不同。