1.

(a)

Call:

lm(formula = y ~ x)

Residuals:

Min	1Q	Median	3Q	Max
-5.1500	-2.2188	0.1625	2.6875	5.5750

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	168.60000	2.65702	63.45	< 2e-16	***
x	2.03437	0.09039	22.51	2.16e-12	***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.234 on 14 degrees of freedom

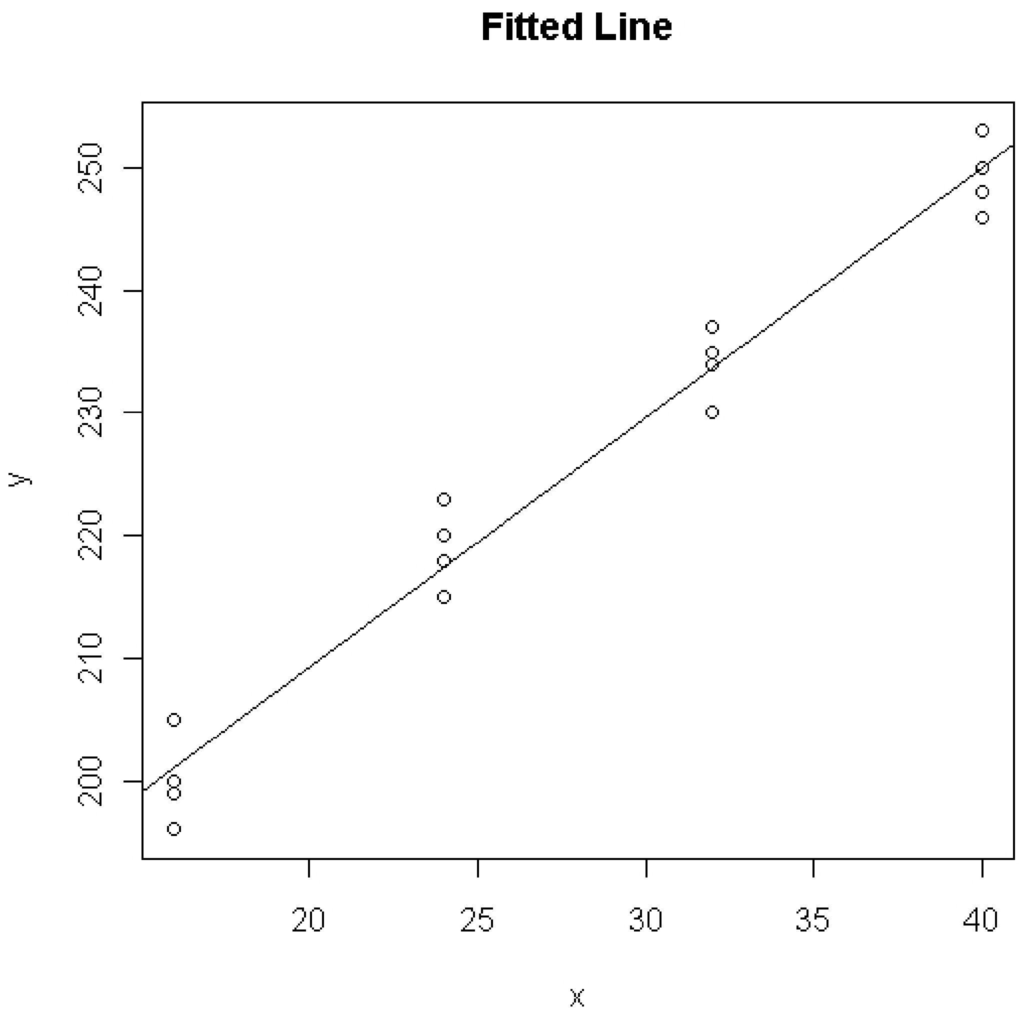
Multiple R-squared: 0.9731, Adjusted R-squared: 0.9712

F-statistic: 506.5 on 1 and 14 DF, p-value: 2.159e-12

Estimated regression function: $\hat{Y} = 168.60000 + 2.03437X$

(b)p-value=2.16e-12 很小，故 reject $\beta_0 = 0$

從圖形來看，還滿適合的，而且 Adjusted R-squared: 0.9712 也很高，代表此 regression line 的解釋效力很高，約有 97%可由此線解釋



(c) $H_0: \beta_1 = 2, H_1: \beta_1 \neq 2$

p-value $= 0.7094445 > 0.01$, do not reject H_0

(d)

	2.5 %	97.5 %
x	1.840500	2.228250

(e)

Lower	Upper
242.4562	257.4938

(f)

Lower	Upper
242.4562	257.4938

(g) $H_0: \beta_1 = 0, H_1: \beta_1 \neq 0$

p-value $= 2.159e\text{-}12 < 0.01$, reject H_0

(h) $R^2 = 0.9731, r = \sqrt{0.9731} = 0.9864583$ (slope>0)

2.

(a) $H^2=(X(X^TX)^{-1}X^T)^2=X(X^TX)^{-1}X^TX(X^TX)^{-1}X^T=X(X^TX)X^T=H$

$(I-H)^2=I-2H+H^2=I-2H+H=I-H$

$HX=X(X^TX)^{-1}X^TX=X$

(b) $A(I-H)=(X^TX)^{-1}X^T(I-X(X^TX)^{-1}X^T)$

$$=(X^TX)^{-1}X^T-(X^TX)^{-1}X^TX(X^TX)^{-1}X^T$$
$$=(X^TX)^{-1}X^T-(X^TX)^{-1}X^TX(X^TX)^{-1}X^T$$
$$=(X^TX)^{-1}X^T-(X^TX)^{-1}X^T=O$$

$(I-H)A^T=(I-X(X^TX)^{-1}X^T)((X^TX)^{-1}X^T)^T$

$$=(I-X(X^TX)^{-1}X^T)X((X^TX)^{-1})^T$$
$$=X((X^TX)^{-1})^T-X(X^TX)^{-1}X^TX((X^TX)^{-1})^T$$
$$=X((X^TX)^{-1})^T-X((X^TX)^{-1})^T=O$$

$H(I-H)=H-H^2=H-H=O$

$(I-H)^TH^T=H(I-H)^T=O^T=O$

3.characteristic polynomial:

$$f(t)=\det(tI-A)=\begin{vmatrix}t-1 & -\rho\\ -\rho & t-1\end{vmatrix}=(t-1)^2-\rho^2=[t-(1+\rho)][t-(1-\rho)]$$

Eigenvalue: $1+\rho, 1-\rho$

Eigenvector:

$t=1+\rho$:

$$A-(1+\rho)I=\begin{bmatrix}-\rho & \rho\\ \rho & -\rho\end{bmatrix}\to\begin{bmatrix}1 & -1\\ 0 & 0\end{bmatrix}\Rightarrow x-y=0$$

取 $\overline{u_1}=\begin{bmatrix}1\\1\end{bmatrix},\overline{e_1}=\dfrac{\overline{e_1}}{|\overline{e_1}|}=\begin{bmatrix}\dfrac{\sqrt{2}}{2}\\ \dfrac{\sqrt{2}}{2}\end{bmatrix}$

$t=1-\rho$:

$$A-(1+\rho)I=\begin{bmatrix}\rho & \rho\\ \rho & \rho\end{bmatrix}\to\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}\Rightarrow x+y=0$$

取 $\overline{u_2}=\begin{bmatrix}1\\-1\end{bmatrix},\overline{e_2}=\dfrac{\overline{e_2}}{|\overline{e_2}|}=\begin{bmatrix}\dfrac{\sqrt{2}}{2}\\ -\dfrac{\sqrt{2}}{2}\end{bmatrix}$

4.

(a)令 $\boldsymbol{Y}=\begin{bmatrix}Y_1\\Y_2\\Y_3\end{bmatrix},\boldsymbol{A}=\begin{bmatrix}1&1&1\\1&-1&0\\1&-1&-1\end{bmatrix}$, 則 $\boldsymbol{W}=\boldsymbol{AY}$

(b)令 $E[Y_1]=\mu_1, E[Y_2]=\mu_2, E[Y_3]=\mu_3$

$$\text{則 } E[\boldsymbol{Y}]=\begin{bmatrix}\mu_1\\\mu_2\\\mu_3\end{bmatrix}\Rightarrow E[\boldsymbol{W}]=E[\boldsymbol{AY}]=\boldsymbol{A}E[\boldsymbol{Y}]=\begin{bmatrix}1&1&1\\1&-1&0\\1&-1&-1\end{bmatrix}\begin{bmatrix}\mu_1\\\mu_2\\\mu_3\end{bmatrix}=\begin{bmatrix}\mu_1+\mu_2+\mu_3\\\mu_1-\mu_2\\\mu_1-\mu_2-\mu_3\end{bmatrix}$$

(c)令 $Cov(Y_i,Y_j)=\sigma_{ij}, i=1,2,3, j=1,2,3$

$$Cov(\boldsymbol{W})=Cov(\boldsymbol{AY})=\boldsymbol{A}Cov(\boldsymbol{Y})\boldsymbol{A}^T=\begin{bmatrix}1&1&1\\1&-1&0\\1&-1&-1\end{bmatrix}\begin{bmatrix}\sigma_1^2&\sigma_{12}&\sigma_{13}\\\sigma_{12}&\sigma_2^2&\sigma_{23}\\\sigma_{13}&\sigma_{23}&\sigma_3^2\end{bmatrix}\begin{bmatrix}1&1&1\\1&-1&-1\\1&0&-1\end{bmatrix}$$

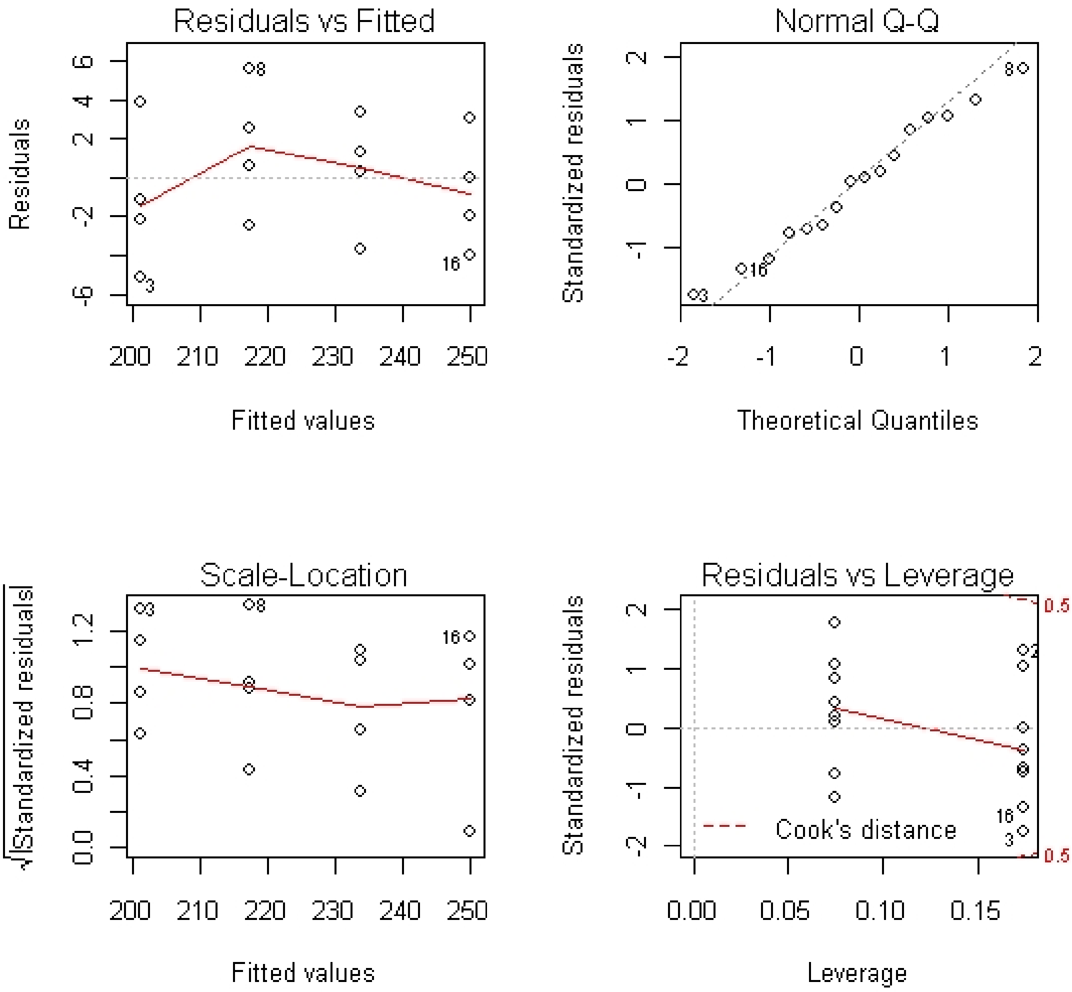
$$=\begin{bmatrix}\sigma_1^2+\sigma_{12}+\sigma_{13}&\sigma_{12}+\sigma_2^2+\sigma_{23}&\sigma_{13}+\sigma_{23}+\sigma_3^2\\\sigma_1^2-\sigma_{12}&\sigma_{12}-\sigma_2^2&\sigma_{13}-\sigma_{23}\\\sigma_1^2-\sigma_{12}-\sigma_{13}&\sigma_{12}-\sigma_2^2-\sigma_{23}&\sigma_{13}-\sigma_{23}-\sigma_3^2\end{bmatrix}\begin{bmatrix}1&1&1\\1&-1&-1\\1&0&-1\end{bmatrix}$$

$$=\begin{bmatrix}\sigma_1^2+\sigma_2^2+\sigma_3^2+2\sigma_{12}+2\sigma_{13}+2\sigma_{23}&\sigma_1^2+\sigma_{13}-\sigma_2^2-\sigma_{23}&\sigma_1^2-\sigma_2^2-2\sigma_{23}-\sigma_3^2\\\sigma_1^2-\sigma_{12}+\sigma_{12}-\sigma_2^2+\sigma_{13}-\sigma_{23}&\sigma_1^2-2\sigma_{12}+\sigma_2^2&\sigma_1^2-2\sigma_{12}+\sigma_2^2-\sigma_{13}+\sigma_{23}\\\sigma_1^2-\sigma_2^2-2\sigma_{23}-\sigma_3^2&\sigma_1^2-2\sigma_{12}-\sigma_{13}+\sigma_2^2+\sigma_{23}&\sigma_1^2-2\sigma_{12}-2\sigma_{13}+\sigma_2^2+2\sigma_{23}+\sigma_3^2\end{bmatrix}$$

5.(a)

i. 由圖形(右上角)看來，第 3, 8, 16 個資料點，是 unusual data points(超過 2 個 standard error)

ii. 由圖形(左上角)看來，第 3, 8, 16 個資料點，是 unusual data points(超過 2 個 standard error)



iii. $h_{ii} - 2\bar{h}$:

[1] -0.075 -0.075 -0.075 -0.075 -0.175 -0.175 -0.175 -0.175 -0.175 -0.175
[11] -0.175 -0.175 -0.075 -0.075 -0.075 -0.075

都是 negative, 故由 leverage 判別並無 outliers

iv. 由圖形(右下角)知，並沒有比例超過很多的值，故由 Cook’s distance 判別並無 outliers.

v. 由圖形(右下角)知，並沒有超過 2 的值，故由 studentized deleted residuals 判別並無 outliers.

(b)

Analysis of Variance Table

Model 1: y ~ x
Model 2: y ~ factor(x)

```
   Res.Df        RSS Df Sum of Sq          F Pr(>F)
1        14 146.425
2        12 128.750    2       17.675 0.8237 0.4622
```

p-value $= 0.4622 > 0.05$, do not reject $H_0: E[Y] = \beta_0 + \beta_1 x$

(c)

Analysis of Variance Table

Model 1: y ~ x
Model 2: y ~ x + I(x^2)

```
   Res.Df        RSS Df Sum of Sq          F Pr(>F)
1        14 146.425
2        13 132.362    1       14.063 1.3812    0.261
```

p-value $= 0.4622 > 0.05$, do not reject $H_0: E[Y] = \beta_0 + \beta_1 x + \beta_2 x^2$

但是 linear regression 的 fitness 比較好

6.(a)

λ	SSE
-0.2	0.733
-0.1	0.3326
0	0.1718
0.1	0.1671
0.2	0.2664

建議可以用 $\lambda = 0$，因為其在 95%的 confidence interval 內

(b)

```
Call:
lm(formula = log(Y) ~ X)

Residuals:
      Min          1Q     Median           3Q          Max
-0.19102 -0.10228    0.01569    0.07716    0.19699

Coefficients:
                        Estimate Std. Error t value Pr(>|t|)
(Intercept)    1.50792       0.06028      25.01 2.22e-12 ***
X                   -0.44993       0.01049    -42.88 2.19e-15 ***
---
Signif. codes:    0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

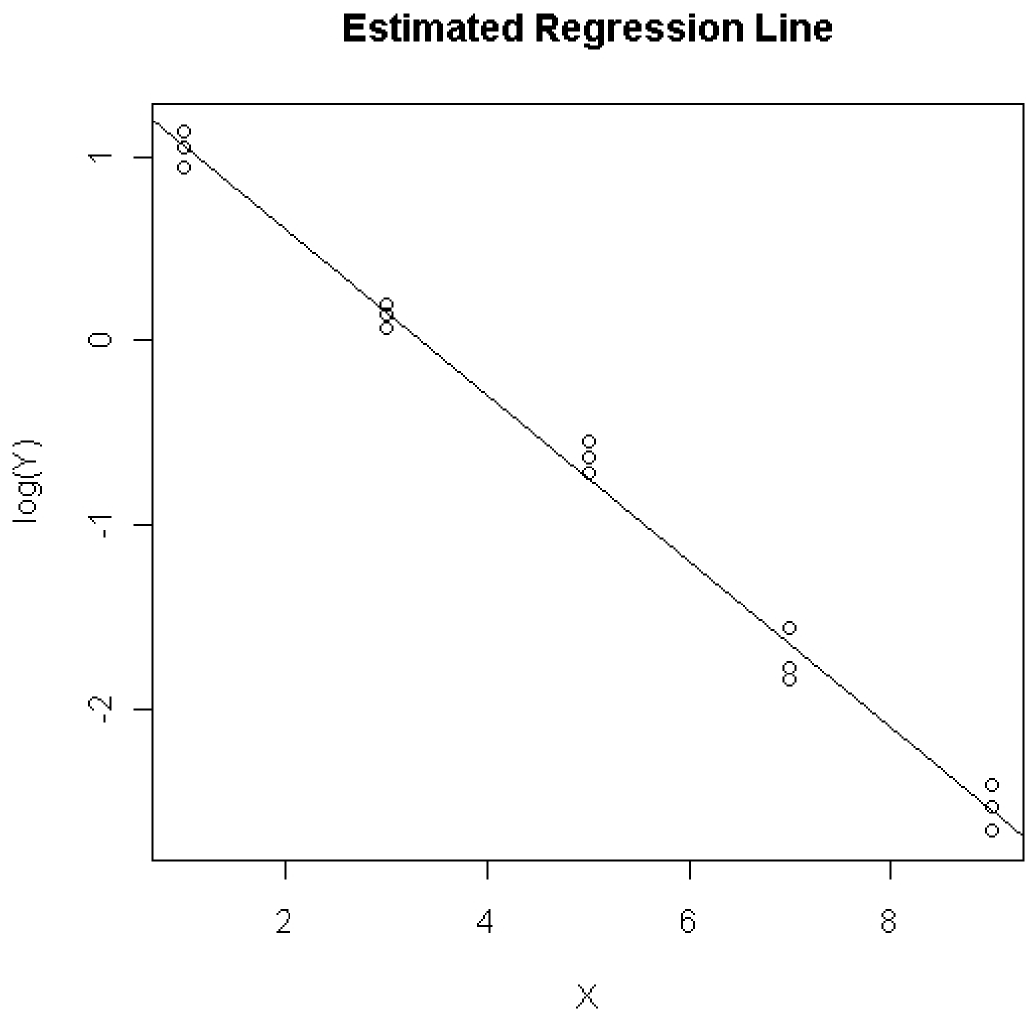
Residual standard error: 0.115 on 13 degrees of freedom

Multiple R-squared: 0.993, Adjusted R-squared: 0.9924

F-statistic: 1838 on 1 and 13 DF, p-value: 2.188e-15

regression function: $\ln\hat{Y} = 1.50792 - 0.44993X$

(c)從圖形來看，似乎沒有 unusual data points



(d)

Call:

lm(formula = Y ~ X)

Residuals:

Min	1Q	Median	3Q	Max
-0.5333	-0.4043	-0.1373	0.4157	0.8487

Coefficients:

	Estimate	Std. Error	t value	Pr(>|t|)	
(Intercept)	2.5753	0.2487	10.354	1.20e-07	***
X	-0.3240	0.0433	-7.483	4.61e-06	***

Signif. codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.4743 on 13 degrees of freedom

Multiple R-squared: 0.8116, Adjusted R-squared: 0.7971

F-statistic: 55.99 on 1 and 13 DF, p-value: 4.611e-06

estimated regression function in the original units: $\hat{Y} = 2.5753 - 0.3240X$