

#48. Since

$$y = \ln \left(\frac{1 + e^x}{1 - e^x} \right) = \ln(1 + e^x) - \ln(1 - e^x),$$

we have

$$y' = \frac{e^x}{1 + e^x} - \frac{-e^x}{1 - e^x} = \frac{2e^x}{(1 + e^x)(1 - e^x)} = \frac{2e^x}{1 - e^{2x}}$$

#53. Since $y = (x^2 - 2x + 2)e^x$, we have $y' = (2x - 2)e^x + (x^2 - 2x + 2)e^x = x^2e^x$.

#97. Let $u = \frac{3}{x}$, then $du = -\frac{3}{x^2}dx$.

$$\int_1^3 \frac{e^{3/x}}{x^2} dx = \int_{3/1}^{3/3} -\frac{1}{3}e^u du = \left[-\frac{1}{3}e^u \right]_3^1 = \frac{e}{3}(e^2 - 1).$$

#99. Let $u = 1 - e^x$, then $du = -e^x dx$.

$$\int e^x \sqrt{1 - e^x} dx = \int -u^{1/2} du = -\frac{2}{3}u^{3/2} + C = -\frac{2}{3}(1 - e^x)^{3/2} + C.$$

#110. Since

$$\frac{dy}{dx} = (e^x - e^{-x})^2 = e^{2x} - 2 + e^{-2x},$$

we have

$$y = \int (e^{2x} - 2 + e^{-2x}) dx = \frac{1}{2}e^{2x} - 2x - \frac{1}{2}e^{-2x} + C.$$

#131. (a) Omit the graph. (Do it by yourself.) Since $f'(x) = \frac{1 - \ln x}{x^2} < 0$ for $e < x$, the function is strictly decreasing on (e, ∞) .

(b) Since $e \leq A < B$, by (a), we have $\frac{\ln A}{A} > \frac{\ln B}{B}$. or $B \ln A > A \ln B$ or $\ln A^B > \ln B^A$. Hence $A^B > B^A$.

(c) Apply (b) by choosing $A = e$ and $B = \pi$.